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On a Multi-Physics Modelling Framework for Thermo-Elastic-Plastic Materials Processing

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Abstract

The present study is devoted to the boundary value problem of thermoelastoplasticity. The new analytical solution of the problem of irregularly heating of the thermo-elastic-plastic hollow cylinder was constructed. The Ishlinsky-Ivlev's and Tresca's yield conditions were used as the plastic potential. The yield stress was assumed linear depending on temperature. The thermal stresses and irreversible strains were computed and compared for different plastic potentials. It is shown that the using of the Ishlinsky-Ivlev's yield condition gives the simpler solution for boundary value problems in the irreversible deformation domains.

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1. Introductory

The calculation of thermal stresses and irreversible (residual) strains is one of the actual problems of modern solid mechanics. The temperature stress computing in a cylindrical symmetry primarily deals with the stress-strain state in the pipes, shafts, couplings and other cylindrical products subjected to intense heat. Calculation of residual stress level, taking into account the plastic flows, for example, in the problems of a hot landing cylindrical bodies [2],

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allows us to correctly calculate the final tightness resulting compound. High temperature gradient can cause irreversible deformation processes in sustainable materials of the shaft components, pipes and disks [3, 4, 5].

The boundary value problem of the cylindrical bodies heating and cooling as previously considered in [1-12]. For example, in [1] the analytical solution was found for the Tresca's yield criteria in problems of the plastic flow. It was shown that the irreversible deformation region composed of several parts. The analytical solution for the central heating of a thin circular plate with Ishlinsky-Ivlev's yield condition depending on temperature was obtained in [4]. It has been found that the assumption of the Tresca's yield leads to physically incorrect results in the irreversible deformation domain. The results obtained in [4] are valid for the case of plane strain condition. Yield strength is constitutive material parameter and strongly depends on temperature. Therefore, such dependence assumed in the thermal plasticity problems allows to obtain more faithful solutions in the plastic domains, as well as to identify the differences in solutions for different yield criteria. In this paper, we solved one-dimensional problem of long hollow cylinder heating by irregular thermal field under Ishlinsky-Ivlev's yield condition.

1.1. Boundary value problem statement. Thermoelastic equilibrium

Consider hollow sustainable material cylinder with inner and outer radii R_1 and R_2 respectively. The strains occurring in cylinder are infinitesimal and compound from reversible (elastic) e_{ij} and irreversible (plastic) p_{ij} parts.

$$d_{rr} = e_{rr} + p_{rr} = u_{r,r}\,,\ d_{\varphi\varphi} = e_{\varphi\varphi} + p_{\varphi\varphi} = \frac{u_r}{r}\,,\ d_{zz} = e_{zz} + p_{zz} = 0. \tag{1}$$

Here u_r is the radial component of the displacement vector, the index after the comma denotes the partial derivative with respect to the corresponding spatial coordinates.

The free thermal expansion conditions in the cylindrical coordinate system are given by:

$$\sigma_{rr}(R_1) = 0\,,\ \sigma_{rr}(R_2) = 0\,. \tag{2}$$

The Duhamel-Neumann stress-strain relations [9] for isotropic materials are read:

$$\begin{aligned}
\sigma_{rr} &= (\lambda + 2\mu)e_{rr} + \lambda(e_{\varphi\varphi} + e_{zz}) - (3\lambda + 2\mu)\Delta,\\
\sigma_{\varphi\varphi} &= (\lambda + 2\mu)e_{\varphi\varphi} + \lambda(e_{rr} + e_{zz}) - (3\lambda + 2\mu)\Delta,\\
\sigma_{zz} &= (\lambda + 2\mu)e_{zz} + \lambda(e_{\varphi\varphi} + e_{rr}) - (3\lambda + 2\mu)\Delta.
\end{aligned} \tag{3}$$

Herein λ, μ denote the constitutive constants (Lame modulus), Δ is the linear thermal expansion strain.

The equilibrium equation and the continuity equation in the cylindrical symmetry case can transformed by:

$$\sigma_{rr,r} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = 0,\ \ d_{\varphi\varphi,r} + \frac{d_{\varphi\varphi} - d_{rr}}{r} = 0. \tag{4}$$

For thermoelastic equilibrium the equation $p_{ij} = 0$ is satisfied along with eq. (4) and the components of stress tensor and displacement vector can be derived:

$$\sigma_{rr} = -\frac{2\omega}{r^2}\int_{R_1}^{r}\Delta(\rho,t)\rho d\rho + A(t) + \frac{B(t)}{r^2}, \quad \sigma_{\varphi\varphi} = \frac{2\omega}{r^2}\int_{R_1}^{r}\Delta(\rho,t)\rho d\rho - 2\omega\Delta(r,t) + A(t) - \frac{B(t)}{r^2},$$

$$\sigma_{zz} = -2\omega\Delta(r,t) + \frac{\lambda A(t)}{(\lambda+\mu)}, \quad u_r = \frac{\omega}{\mu r}\int_{R_1}^{r}\Delta(\rho,t)\rho d\rho + \frac{A(t)r}{2(\lambda+\mu)} - \frac{B(t)}{2\mu r}, \quad \omega = \frac{\mu(3\lambda+2\mu)}{(\lambda+2\mu)}. \tag{5}$$

Here $A(t)$, $B(t)$ are the time dependent function, obtained from the boundary conditions (2).

1.2. Plastic Flow

Assume that a uniform cylindrical material is subjected to thermal expansion $\Delta = const$. For a certain value $\Delta = \Delta_p$ the entire volume of cylinder is exposed to plastic flow, under conditions:

$$\sigma_{rr} - \sigma_{zz} = 2k(\Delta_p), \quad \sigma_{\varphi\varphi} - \sigma_{zz} = 2k(\Delta_p). \tag{6}$$

Herein k is yield stress. In general, one depends on temperature. Note that radial and tangential components of stress tensor are vanished due to the free thermal expansion. Thus, irreversible deformation in this case is coupled with increasing of the stress tensor component σ_{zz} in consequence of constrictions on thermal expansion along the coordinate z. For stresses and displacements in plastic flow domain we derive using the eqs. (4) and (6) the following equations:

$$\sigma_{zz} = 2k(\Delta_p), \quad \sigma_{rr} = \sigma_{\varphi\varphi} = 0, \quad u_{rr} = r\left(\frac{3\Delta_p}{2} - \frac{k(\Delta_p)}{(3\lambda+2\mu)}\right). \tag{7}$$

2. Tresca's Solution under Uneven Thermal Expansion

Assume that the temperature of the inner surface of the cylinder is slowly increased from the initial T_0 to a maximum temperature t, in which the process of irreversible deformation occurs. The function of thermal expansion in this case is the exact solution of the stationary heat equation [1]:

$$\Delta(r,t) = \frac{\alpha(t-T_0)\ln(r/R_2)}{\ln(R_1/R_2)}, \tag{8}$$

where α denote linear thermal expansion coefficient.

Tresca's yield condition for the problem is given by [10, 11]:

$$\max\{|\sigma_{rr} - \sigma_{\varphi\varphi}|, |\sigma_{\varphi\varphi} - \sigma_{zz}|, |\sigma_{zz} - \sigma_{rr}|\} = 2k(r,t), \qquad k(r,t) = k_0(1-\beta\Delta(r,t)) \tag{9}$$

Here k_0 is the yield strength at referential temperature, β is the constitutive constant which specifies a reduction rate of yield strength with increasing temperature.

The increasing of parameter t leads to the satisfaction of the conditions (9) at inner cylinder surface as

$$\sigma_{rr} - \sigma_{zz} = 2k(R_1, t_1). \tag{10}$$

Under condition $t > t_1$ the irreversible deformation domain $R_1 < r < a_1$ is formed in the vicinity of the inner surface. The elastoplastic surface $a_1(t)$ separates the plastic flow domain from the thermoelastic deformation one $a_1 < r < R_2$. The following equations for the plastic strains can be derived from condition (10) and the associated plastic flow rule

$$p_{rr} + p_{zz} = 0, \quad p_{\varphi\varphi} = 0. \qquad (11)$$

We simply find the differential equation for the radial displacement from the system of equations (3), (4), (10), and (11):

$$(ru_{r,r})_{,r} - \frac{\eta^2 u_r}{r} + \frac{(rk)_{,r}}{(\lambda+\mu)} - \frac{\gamma r \Delta_{,r}}{\mu} = 0, \quad \gamma = \frac{\mu(3\lambda+2\mu)}{(\lambda+\mu)}, \quad \eta = \sqrt{\frac{(\lambda+2\mu)}{(\lambda+\mu)}}. \qquad (12)$$

The solution of equation (12) is

$$u_r = \frac{\gamma}{2\mu\eta}\left(\frac{(\eta+1)}{r^\eta}\int_{R_1}^{r}\rho^\eta \Delta(\rho,t)d\rho + (\eta-1)r^\eta\int_{R_1}^{r}\frac{\Delta(\rho,t)}{\rho^\eta}d\rho\right) -$$
$$-\frac{1}{2(\lambda+\mu)}\left(\frac{1}{r^\eta}\int_{R_1}^{r}\rho^\eta k(\rho,t)d\rho + r^\eta\int_{R_1}^{r}\frac{k(\rho,t)}{\rho^\eta}d\rho\right) + C(t)r^\eta + \frac{D(t)}{r^\eta},$$
$$p_{rr} = \frac{\eta}{4(\lambda+\mu)}\left(\frac{1}{r^{\eta+1}}\int_{R_1}^{r}\rho^\eta k(\rho,t)d\rho + r^{\eta-1}\int_{R_1}^{r}\frac{k(\rho,t)}{\rho^\eta}d\rho\right) + \frac{\gamma\Delta(r,t)}{2\mu} - \frac{\eta^2 k(r,t)}{2\mu} +$$
$$+\frac{\gamma}{4\mu}\left(r^{\eta-1}\int_{R_1}^{r}\frac{k(\rho,t)}{\rho^\eta}d\rho - \frac{(\eta+1)}{r^{\eta+1}}\int_{R_1}^{r}\rho^\eta\Delta(\rho,t)d\rho\right) + \frac{\eta r^{\eta-1}C(t)}{2} - \frac{\eta D(t)}{2r^{\eta+1}}, \qquad (13)$$

Thermal stresses in plastic flow domain $R_1 < r < a_1$ one can compute using eq. (13).

$$\sigma_{rr} = \sigma_{zz} + 2k(r,t) = \frac{\gamma}{2\eta\mu}\left((\eta-1)\nu_1 r^{\eta-1}\int_{R_1}^{r}\frac{\Delta(\rho,t)}{\rho^\eta}d\rho - \frac{(\eta+1)\nu_2}{r^{\eta+1}}\int_{R_1}^{r}\rho^\eta\Delta(\rho,t)d\rho\right) -$$
$$-\frac{1}{2(\lambda+\mu)}\left(\nu_1 r^{\eta-1}\int_{R_1}^{r}\frac{k(\rho,t)}{\rho^\eta}d\rho - \frac{\nu_2}{r^{\eta+1}}\int_{R_1}^{r}\rho^\eta k(\rho,t)d\rho\right) + \nu_1 r^{\eta-1}C(t) - \frac{\nu_2 D(t)}{r^{\eta+1}},$$
$$\sigma_{\varphi\varphi} = \frac{\gamma}{2\eta\mu}\left((\eta-1)\upsilon_1 r^{\eta-1}\int_{R_1}^{r}\frac{\Delta(\rho,t)}{\rho^\eta}d\rho - \frac{(\eta+1)\upsilon_2}{r^{\eta+1}}\int_{R_1}^{r}\rho^\eta\Delta(\rho,t)d\rho\right) - \gamma\Delta(r,t) - \frac{\lambda k(r,t)}{(\lambda+\mu)} -$$
$$-\frac{1}{2(\lambda+\mu)}\left(\upsilon_1 r^{\eta-1}\int_{R_1}^{r}\frac{k(\rho,t)}{\rho^\eta}d\rho - \frac{\upsilon_2}{r^{\eta+1}}\int_{R_1}^{r}\rho^\eta k(\rho,t)d\rho\right) + \upsilon_1 r^{\eta-1}C(t) - \frac{\upsilon_2 D(t)}{r^{\eta+1}}.$$
$$\nu_1 = (\lambda+\eta\lambda+\eta\mu), \quad \nu_2 = (\eta\lambda-\lambda+\eta\mu), \quad \upsilon_1 = (\lambda+\eta\lambda+2\mu), \quad \upsilon_2 = (\lambda-\eta\lambda+2\mu). \qquad (14)$$

The unknown functions in eqs. (5), (13), and (14) are found from the boundary conditions (2) and the conditions of radial stresses and displacements continuity on the elastic-plastic border. The elastic-plastic border value a_1 for a given parameter value t is found by numerical solution of the equation $p_{rr}(a_1,t) = 0$.

The resulting solutions system (5), (13), and (14) is valid in a certain temperature range $\Delta_1 < t < \Delta_2$, because at time $t = t_2$ two conditions are simultaneously satisfied on inner cylinder surface:

$$\sigma_{rr} - \sigma_{zz} = 2k(R_1, t_2), \quad \sigma_{rr} - \sigma_{\varphi\varphi} = 2k(R_1, t_2). \tag{15}$$

This fact means transformation of the cylinder material in a completely plastic state. At the time $t > t_2$ the elastoplastic border a_2 separates the full plasticity domain $R_1 < r < a_2$ from the plastic flow one $a_2 < r < a_1$. The stresses in full plasticity domain are found by integrating of equilibrium equation taking into account the conditions (15):

$$\sigma_{rr} = -2\int_{R_1}^{r} \frac{k(\rho,t)}{\rho} d\rho + E(t), \quad \sigma_{\varphi\varphi} = \sigma_{zz} = -2\int_{R_1}^{r} \frac{k(\rho,t)}{\rho} d\rho - 2k(r,t) + E(t). \tag{16}$$

We derive the displacement in the domain $R_1 < r < a_2$ assuming plastic incompressibility and using (3) and (16):

$$u_r = -\frac{1}{(3\lambda + 2\mu)} \left(\frac{1}{r} \int_{R_1}^{r} k(\rho,t)\rho d\rho + 3r \int_{R_1}^{r} \frac{k(\rho,t)}{\rho} d\rho - \frac{3rE(t)}{2} \right) + \frac{3}{r} \int_{R_1}^{r} \Delta(\rho,t)\rho d\rho + \frac{F(t)}{r}. \tag{17}$$

The unknown function is necessary to re-determine by the boundary conditions (2) and the stress-strain state parameter continuity.

The plastic strains are calculated by:

$$p_{rr} = \frac{1}{(3\lambda + 2\mu)} \left(\frac{1}{r^2} \int_{R_1}^{r} k(\rho,t)\rho d\rho - \int_{R_1}^{r} \frac{k(\rho,t)}{\rho} d\rho + \frac{E(t)}{2} \right) - \frac{2k(r,t)}{\gamma} - \frac{3}{r^2} \int_{R_1}^{r} \Delta(\rho,t)\rho d\rho + 2\Delta(r,t) - \frac{F(t)}{r^2},$$

$$p_{\varphi\varphi} = -\frac{1}{(3\lambda + 2\mu)} \left(\frac{1}{r^2} \int_{R_1}^{r} k(\rho,t)\rho d\rho + \int_{R_1}^{r} \frac{k(\rho,t)}{\rho} d\rho - \frac{E(t)}{2} \right) + \frac{k(r,t)}{\gamma} + \frac{3}{r^2} \int_{R_1}^{r} \Delta(\rho,t)\rho d\rho - \Delta(r,t) + \frac{F(t)}{r^2},$$

$$p_{zz} = \frac{1}{(3\lambda + 2\mu)} \left(2\int_{R_1}^{r} \frac{k(\rho,t)}{\rho} d\rho - E(t) \right) + \frac{k(r,t)}{\gamma} - \Delta(r,t). \tag{18}$$

The elastic-plastic borders values are computed by numerical solution of the equations $p_{rr}(a_1, t) = 0$ and $p_{\varphi\varphi}(a_2, t) = 0$.

3. Ishlinsky-Ivlev's Solution under Uneven Thermal Expansion

Ishlinsky-Ivlev's yield condition [11, 12] is given by:

$$\max\{|\sigma_{rr} - \sigma|, |\sigma_{\varphi\varphi} - \sigma|, |\sigma_{zz} - \sigma|\} = \frac{4}{3}k(r,t), \quad \sigma = \frac{\sigma_{rr} + \sigma_{\varphi\varphi} + \sigma_{zz}}{3}. \tag{19}$$

At a certain value of temperature $t = T_3$ on the inner cylinder surface the following condition will satisfied

$$2\sigma_{rr} - \sigma_{zz} - \sigma_{\varphi\varphi} = 4k(R_1, T_3), \tag{20}$$

This fact means the irreversible deformation domain occurrence is $R_1 < r < a$. We can derive the equation for plastic strains from eq. (20) and the associated plastic flow rule.

$$p_{rr} + 2p_{\varphi\varphi} = 0, \quad p_{\varphi\varphi} = p_{zz}. \tag{21}$$

The differential equation for the radial component of the displacement vector using eqs. (1), (3), (4), (20), and (21) is obtained:

$$(ru_{r,r})_{,r} - \chi^2 \frac{u_r}{r} + \frac{2(3k + 2rk_{,r})}{(3\lambda + 2\mu)} - 3r\Delta_{,r} = 0, \quad \chi = \sqrt{\frac{(3\lambda + 5\mu)}{(3\lambda + 2\mu)}}. \tag{22}$$

Integrating the equation (22) we find the function of radial displacement in the domain $R_1 < r < a$:

$$u_r = -\frac{1}{\chi(3\lambda + 2\mu)}\left(r^{\chi}(2\chi + 1)\int_{R_1}^{r} \frac{k(\rho,t)}{\rho^{\chi}} d\rho + \frac{(2\chi - 1)}{r^{\chi}} \int_{R_1}^{r} \rho^{\chi} k(\rho,t) d\rho \right) + \\ + \frac{3}{2\chi}\left(r^{\chi}(\chi - 1)\int_{R_1}^{r} \frac{\Delta(\rho,t)}{\rho^{\chi}} d\rho + \frac{(\chi + 1)}{r^{\chi}} \int_{R_1}^{r} \rho^{\chi} \Delta(\rho,t) d\rho \right) + r^{\chi} X(t) + \frac{Y(t)}{r^{\chi}}. \tag{23}$$

And the radial plastic strain:

$$p_{rr} = \frac{(1 - 4\chi^2)}{3\chi(3\lambda + 2\mu)}\left(r^{\chi-1}\int_{R_1}^{r} \frac{k(\rho,t)}{\rho^{\chi}} d\rho - \frac{1}{r^{\chi+1}} \int_{R_1}^{r} \rho^{\chi} k(\rho,t) d\rho \right) - \frac{2k(r,t)}{\omega} + 2\Delta(r,t) + \frac{1}{3}(2\chi - 1) r^{\chi-1} X(t) + \\ + \frac{1}{2\chi}\left((2\chi - 1)(\chi - 1) r^{\chi-1}\int_{R_1}^{r} \frac{\Delta(\rho,t)}{\rho^{\chi}} d\rho - \frac{(2\chi + 1)(\chi + 1)}{r^{\chi+1}} \int_{R_1}^{r} \rho^{\chi} \Delta(\rho,t) d\rho \right) - \frac{(2\chi + 1) Y(t)}{3 r^{\chi+1}}. \tag{24}$$

On the basis of the found functions (23) and (24) the stresses are calculated by:

$$\sigma_{rr} = \frac{1}{3\chi}\left(r^{\chi-1}(\chi + 1)(2\chi + 1)\int_{R_1}^{r} \frac{k(\rho,t)}{\rho^{\chi}} d\rho - \frac{(\chi - 1)(2\chi - 1)}{r^{\chi+1}} \int_{R_1}^{r} \rho^{\chi} k(\rho,t) d\rho \right) + \\ + (3\lambda + 2\mu)\left(\frac{(\chi^2 - 1)}{2\chi}\left(r^{\chi-1}\int_{R_1}^{r} \frac{\Delta(\rho,t)}{\rho^{\chi}} d\rho - \frac{1}{r^{\chi+1}} \int_{R_1}^{r} \rho^{\chi} \Delta(\rho,t) d\rho \right) + \frac{1}{3}\left((\chi + 1) r^{\chi-1} X(t) - \frac{(\chi - 1) Y(t)}{r^{\chi+1}} \right) \right), \\ \sigma_{\varphi\varphi} = \frac{1}{3\chi(3\lambda + 2\mu)}\left(\frac{(2\chi - 1)\xi_2}{r^{\chi+1}} \int_{R_1}^{r} \rho^{\chi} k(\rho,t) d\rho - r^{\chi-1}(2\chi + 1)\xi_1 \int_{R_1}^{r} \frac{k(\rho,t)}{\rho^{\chi}} d\rho \right) - 2k(r,t) + \\ + \frac{1}{2\chi}\left((\chi - 1)\xi_1 r^{\chi-1}\int_{R_1}^{r} \frac{\Delta(\rho,t)}{\rho^{\chi}} d\rho - \frac{(\chi + 1)\xi_2}{r^{\chi+1}} \int_{R_1}^{r} \rho^{\chi} \Delta(\rho,t) d\rho \right) + \frac{1}{3}\left(\xi_1 r^{\chi-1} X(t) - \frac{\xi_2 Y(t)}{r^{\chi+1}} \right), \\ \sigma_{zz} = 2\sigma_{rr}(r,t) - \sigma_{\varphi\varphi}(r,t) - 4k(r,t), \\ \xi_1 = 3\lambda(\chi + 1) + \mu(2\chi + 5), \quad \xi_2 = 3\lambda(\chi - 1) + \mu(2\chi - 5). \tag{25}$$

Unknown functions $A(t)$, $B(t)$, $X(t)$, and $Y(t)$, in eqs. (5), (23), and (24) are simply derived by the boundary conditions (2) and the stress-strain state parameter continuity. The elastic-plastic border value are found from the radial plastic strain vanishing $p_{rr}(a,t)=0$.

Discussion

The following constitutive and material constants were used for numerical calculations: $k_0 = 210\times10^6 Pa$, . $\Delta(R_1)=4.25\times10^{-3}$, $\beta=94.12$, $\lambda=9.12\times10^{10}Pa$, $\mu=4.29\times10^{10}Pa$, $R_2=0.2m$. The Figure 1 shows the results of thermal stress calculations for each of the yield conditions. The level of thermal expansion was chosen so as to exclude the possibility of the plastic flow occurrence in the vicinity of the outer cylinder surface. Note that the difference between the stress level in the plastic flow domain for both yield conditions is smaller with increasing temperature gradient on the inner surface. The difference between the stresses in the irreversible deformation domain may become significant for some ratio values. Obviously, the process of calculating the stress-strain state parameters within the Ishlinsky-Ivlev`s yield condition is simpler because, there is no need to consider the full plasticity possibility in the elastic and unloading domains. This fact is undoubtedly become useful in more sophisticated calculations, such as non-stationary thermal treatment because this can reduce the number of plastic flow domains.

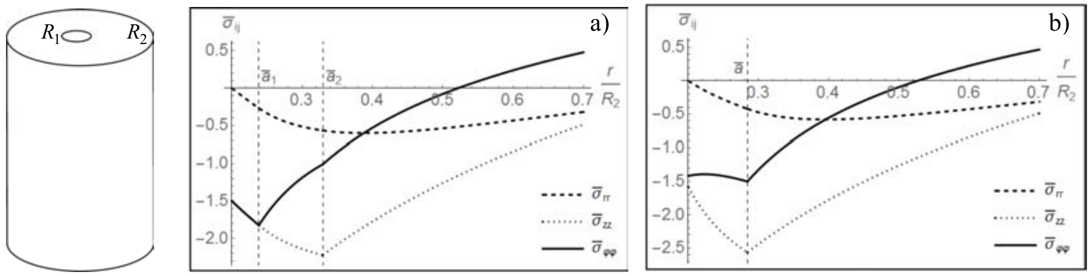

Fig. 1. Thermal Stresses: a) Tresca's yield condition; b) Ishlinsky-Ivlev`s yield condition. $\bar{\sigma}_{ij}=\sigma_{ij}/k_0$, $\bar{a}_i=a/R_2$.

Conclusion

Residual transient thermal stresses during the heat transfer process between the hot and cold cylinders have been calculated and graphically analyzed. It had been shown that in some cases, the final level of the residual stress was mainly dependent on the size of the considered cylindrical parts, the initial temperature difference and the thermal conductivity. As a consequence of such transient thermal stresses distributions did not have a significant impact on the final stress-strain state of the deformable thermo-elastic-plastic material. The two yield criteria in the derivation of solutions for non-linear thermal response of a cylinder have been compared. The original analytical results have been obtained. This is more significant in the prediction of residual stress profiles.

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