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TECHNOLOGICAL THERMAL STRESSES IN THE SHRINK FITTING OF CYLINDRICAL BODIES WITH CONSIDERATION OF PLASTIC FLOWS

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Abstract: This paper presents a solution of a sequence of one-dimensional boundary-value problems of thermal stresses defining the elastic–plastic deformation processes used in the shrink fitting of cylindrical bodies. The initiation and development of plastic flow in the materials of the assembly elements are studied taking into account the temperature dependence of the yield stress of these materials. During temperature equalization, the flow can slow down, followed by unloading and formation of a residual stress field providing tension. The conditions of formation and motion of the boundaries of the elastic and plastic states in plastic flow and during unloading are determined.

Keywords: elasticity, plasticity, thermal stresses, shrink fitting, residual stresses.

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INTRODUCTION

Until recently, it has been recommended by regulatory documents to use computational methods for shrink fitting based on the theory of elastic thermal stresses [1, 2]. The plastic properties of the assembly materials have not been taken into account, so that the initiation and development of plastic flow have not been investigated. In the middle of the 20th century, Bland found that at significant temperature gradients, it is thermal stresses in the tube material under thermomechanical loading that can cause plastic flow [3]. Significant reduction in the material yield stress with increasing temperature should also be taken into account. The results of solution of one-dimensional contact problems obtained using the finite element method are presented in [4, 5]. The temperature dependence of the yield stress was assumed to be linear.

In this paper, exact solutions of a sequence of deformation boundary-value problems of the theory of thermal stresses characterizing the assembly process are obtained using the results of numerical calculations obtained by solving contact boundary-value problems for the non-stationary heat-conduction equation. Emphasis is placed on determining the place and time of initiation of the flow, its development and deceleration during unloading due to cooling of the material, as well as identifying the laws of motion of the boundaries of the regions of elastic and plastic flows. In accordance with well-known experimental data [6, 7], a quadratic dependence of the yield stress on temperature is used.

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FORMULATION OF THE PROBLEM. TEMPERATURE DISTRIBUTION

Suppose that a cylindrical tube heated to a temperature T_* and having dimensions $R_1 \leqslant r \leqslant R_2$ is shrink-fitted onto a tube with dimensions $R_0 \leqslant r \leqslant R_1$ whose temperature is equal to the room temperature T_0. Neglecting the relationship between the thermal and deformation processes and the influence of edge effects, we obtain a one-dimensional problem of the theory of thermal stresses. For a non-stationary temperature field in the materials of the connected cylindrical bodies in cylindrical coordinates (r, φ, z), the heat-conduction equation is written as

$$T_{,t} = a(T_{,rr} + r^{-1}T_{,r}), \tag{1}$$

where the subscript after the comma denotes differentiation with respect to the corresponding variable, $T(r,t)$ is the current temperature, and a is the thermal diffusivity.

Let us define the boundary and initial conditions. The initial conditions are

$$T(r,0) = \begin{cases} T_0, & R_0 \leqslant r \leqslant R_1, \\ T_*, & R_1 \leqslant r \leqslant R_2. \end{cases}$$

At $r = R_0$ and $r = R_2$, the heated body comes in contact with the environment:

$$T_{,r}\Big|_{r=R_0,R_2} = \chi T_0. \tag{2}$$

Here the constant χ is the heat-transfer coefficient of the heated material into the environment. On the surface $r = R_1$, there is thermal contact:

$$[T_{,r}]\Big|_{r=R_1} = 0, \qquad [T]\Big|_{r=R_1} = 0. \tag{3}$$

Here the square brackets denote the discontinuity of the quantity on the surface $r = R_1$, i.e., $[T] = T^+ - T^-$, where T^+ and T^- are the surface temperatures of the inner and outer tubes, respectively. In (3), as in (1), the simplifying assumption that the assembly elements are made of the same materials.

The solution of Eq. (1) with initial and boundary conditions (2), (3) can be obtained both analytically [8, 9] and numerically [10]. Below we will assume that it is known.

REVERSIBLE DEFORMATION

We assume that the time $t = 0$ is the time of shrink-fitting of the cylindrical bodies. At $t > 0$, the temperature distribution in both parts of the assembly is considered known. The corresponding thermal stresses should be determined using the Duhamel–Neumann law [11]

$$\sigma_{ij} = (\lambda e_{kk} - 3\alpha K\theta)\delta_{ij} + 2\mu e_{ij}, \qquad \theta = T - T_0. \tag{4}$$

Here σ_{ij} and e_{ij} are the components of the stress and reversible (elastic) strain tensor, λ, μ, and $K = (\lambda + 2\mu/3)$ are elasticity characteristics, α is the coefficient of linear thermal expansion of the isotropic material of the assembly, and δ_{ij} are the unit tensor components (Kronecker symbols). In (4), rectangular Cartesian coordinates and the rule of summation over repeated indices are used. Along with reversible strains e_{ij}, the assembly material can undergo irreversible (plastic) strains p_{ij}. In view of the smallness of the total strains d_{ij}, they can be expressed as the sum

$$d_{ij} = e_{ij} + p_{ij} = 0.5(u_{i,j} + u_{j,i}), \tag{5}$$

where u_i are the displacement vector components. Plastic strain is accumulated in the material only at stresses satisfying the yield condition $f(\sigma_{ij}) = 0$. According to the Mises maximum principle, the equation of the surface $f(\sigma_{ij}) = 0$ in the stress space plays the role of the plastic potential, which satisfies the associated plastic flow law

$$dp_{ij} = d\lambda\, \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}}, \qquad d\lambda > 0. \tag{6}$$

As the loading surface we use Tresca's prism, which is given by the equation [12]

$$\max |\sigma_i - \sigma_j| = 2k. \tag{7}$$

Here σ_i are the principal values of the stress tensor and $k = k(\theta)$ is the yield stress of the material. The temperature dependence of the yield stress is given by

$$k(r,t) = k(\theta(r,t)) = k_0((T_p - T_0)^{-2}(T_p - T(r,t))^2) \tag{8}$$

(k_0 is the yield stress of the material at room temperature T_0 and T_p is the melting temperature of the material). Relations (4)–(8) and the equilibrium equation form a closed system of equations for calculating the thermal stress from a known temperature distribution. At times close to the time of shrink-fitting ($t = 0$), the materials of the assembly elements are elastically deformed. Then, from (4) the stresses are expressed as

$$\sigma_r^{(v)} = w u_{r,r}^{(v)} + \lambda r^{-1} u_r^{(v)} - m\theta_v, \qquad \sigma_\varphi^{(v)} = \lambda u_{r,r}^{(v)} + w r^{-1} u_r^{(v)} - m\theta_v,$$
$$\sigma_z^{(v)} = \lambda(r^{-1} u_r^{(v)} + u_{r,r}^{(v)}) - m\theta_v, \qquad w = \lambda + 2\mu, \qquad m = 3\alpha K, \tag{9}$$

where $\sigma_r^{(v)}$, $\sigma_\varphi^{(v)}$, and $\sigma_z^{(v)}$ are the principal values of the stress tensor (the superscript $v = 1$ corresponds to the inner cylinder, and $v = 2$ to the outer cylinder), $\theta_v(r,t) = T_v(r,t) - \tilde{T}_v$, and $\tilde{T}_1 = T_0$ and $\tilde{T}_2 = T_*$ are the initial temperatures of the assembly elements. The radial stress $\sigma_r^{(v)}$ and circumferential stress $\sigma_\varphi^{(v)}$ must satisfy the equilibrium equation

$$\sigma_{r,r}^{(v)} + r^{-1}(\sigma_r^{(v)} - \sigma_\varphi^{(v)}) = 0. \tag{10}$$

For a certain temperature distribution at time $t > 0$, the stresses and displacements are calculated by integrating system (9), (10):

$$u_r^{(v)}(r,t) = r w^{-1} F_v(r_v, r, t) + r C_{v1}(t) + r^{-1} C_{v2}(t),$$
$$\sigma_r^{(v)}(r,t) = -2\mu w^{-1} F_v(r_v, r, t) + 2g C_{v1}(t) - 2\mu r^{-2} C_{v2}(t),$$
$$\sigma_\varphi^{(v)}(r,t) = 2\mu w^{-1} F_v(r_v, r, t) - 2m\mu w^{-1}\theta_v(r,t) + 2g C_{v1}(t) + 2\mu r^{-2} C_{v2}(t), \tag{11}$$
$$\sigma_z^{(v)}(r,t) = 2\lambda C_{v1}(t) - 2m\mu w^{-1}\theta_v(r,t),$$
$$F_v(r_v, r, t) = m r^{-2} \int_{r_v}^{r} \theta_v(\rho, t)\rho\, d\rho, \qquad r_v = R_{v-1}, \quad g = \lambda + \mu.$$

The unknown functions $C_{v1}(t)$ and $C_{v2}(t)$ are determined by solving the system of linear equations following from the boundary conditions including the continuity conditions for the radial stresses and displacements at the interface between the joined parts [$\sigma_r^{(1)}(R_1,t) = \sigma_r^{(2)}(R_1,t)$ and $u_r^{(1)}(R_1,t) = u_r^{(2)}(R_1,t)$] and the zero stress condition on the free surfaces of the tubes [$\sigma_r^{(1)}(R_0,t) = 0$ and $\sigma_r^{(2)}(R_2,t) = 0$]. Because the expression defining the functions $C_{v1}(t)$ and $C_{v2}(t)$ is length and is not presented here.

PLASTIC FLOW

The Tresca condition $\sigma_r^{(2)} - \sigma_\varphi^{(2)} = -2k_2$ is first hold on the contact surface of $r = R_1$ in the material of the outer tube. At some time $t = t_1$ ($t_1 > 0$), on the contact surface $r = R_1$ a region of plastic flow $R_1 \leqslant r \leqslant n(t)$ is formed [$r = n(t)$ is the moving boundary of the regions of elastic deformation and plastic flow]. In this region, both reversible and irreversible strains develop. Relations (9) and (10) for the flow region are written as

$$\sigma_r^{(2)} = w(u_{r,r}^{(2)} - p_r^{(2)}) + \lambda(r^{-1}u_r^{(2)} - (p_\varphi^{(2)} + p_z^{(2)})) - m\theta_2,$$
$$\sigma_\varphi^{(2)} = \lambda(u_{r,r}^{(2)} - (p_r^{(2)} + p_z^{(2)})) + w(r^{-1}u - p_\varphi^{(2)}) - m\theta_2, \tag{12}$$
$$\sigma_z^{(2)} = \lambda(r^{-1}u_r^{(2)} + u_{r,r}^{(2)} - (p_\varphi^{(2)} + p_r^{(2)})) - w p_z^{(2)} - m\theta_2;$$

$$\sigma_{r,r}^{(2)} - 2k_2 r^{-1} = 0. \tag{13}$$

Adding the plastic incompressibility condition following from (6) and (7) $p_\varphi^{(2)} + p_r^{(2)} = 0$, $p_z^{(2)} = 0$ to relations (12) and (13), we obtain a system of equations which describes the stress–strain state in the region of plastic flow. The solution of this system of equations has the form

$$
\begin{gathered}
\tilde{u}_r^{(2)}(r,t) = rg^{-1}G(R_1,r,t) + rg^{-1}F_2(R_1,r,t) + C_{23}(t)r + r^{-1}C_{24}(t),\\
\tilde{\sigma}_r^{(2)}(r,t) = G(R_1,r,t) + 2gC_{23}(t),\\
\tilde{\sigma}_\varphi^{(2)}(r,t) = G(R_1,r,t) + 2gC_{23}(t) + 2k_2(r,t),\\
\tilde{\sigma}_z^{(2)}(r,t) = \lambda g^{-1}(G(R_1,r,t) + k_2(r,t)) + 2\lambda C_{23}(t) - m\mu g^{-1}\theta_2(r,t),\\
p_r^{(2)}(r,t) = 0.5mg^{-1}\theta_2(r,t) - g^{-1}F_2(R_1,r,t) + w(\mu g)^{-1}k_2(r,t) - r^{-2}C_{24}(t),\\
G(r_0,r,t) = \int_{r_0}^{r} \rho^{-1}k_2(\rho,t)\,d\rho, \qquad g = \lambda + \mu,
\end{gathered}
\tag{14}
$$

where the tilde above the stress and displacement components implies that they correspond to the plastic flow region.

Thus, at $t > t_1$, the assembly material can be divided into three regions and in the regions $R_0 \leqslant r \leqslant R_1$ and $n(t) \leqslant r \leqslant R_2$, the material is elastically deformed. In the region $R_1 \leqslant r \leqslant n(t)$, irreversible strain is accumulated due to the presence plastic flow. In the first two regions, the thermoelastic strain parameters are given by relations (11), and in flow regions, by relations (14). The functions arising from the integration should be determined again from the boundary conditions on the free surfaces $r = R_0$ and $r = R_2$, on the contact surface $\sigma_r^{(1)}(R_1,t) = \tilde{\sigma}_r^{(2)}(R_1,t)$ and $u_r^{(1)}(R_1,t) = \tilde{u}_r^{(1)}(R_1,t)$ and on the elastic–plastic boundary $\tilde{\sigma}_r^{(2)}(n,t) = \sigma_r^{(2)}(n,t)$ and $\tilde{u}_r^{(2)}(n,t) = u_r^{(2)}(n,t)$. Furthermore, it is necessary to determine the function $n(t)$ which specifies the position of the cylindrical surface at the current time t. For this, we use condition (7) and the condition of zero plastic deformation $[p_r(n,t) = 0]$.

The resulting solution is valid only up to a certain time $t = t_2$ ($t_2 > t_1$), which is due to the formation and development of a new region of plastic flow in the inner tube material. This region develops from the free surface $r = R_0$, and in this region $\sigma_r^{(1)} - \sigma_z^{(1)} = 2k_1$. It should be noted that in the calculation, this is the case if the inner tube is thin enough ($R_1 - R_0 \ll R_2 - R_1$). Consequently, for $t > t_2$, there is also an region of plastic current $R_0 \leqslant r \leqslant m(t)$. The relations between the stresses and displacements in this region taking into account plastic incompressibility $p_z^{(1)} + p_r^{(1)} = 0$ and $p_\varphi^{(1)} = 0$ follow from formulas (12) and (10) in which superscript 2 is replaced by superscript 1:

$$
\begin{gathered}
\tilde{u}_r^{(1)}(r,t) = (2g)^{-1}[(h^{-1}+1)M_{-h}(R_0,r,t) - (h^{-1}-1)M_h(R_0,r,t) - N_h(R_0,r,t) - N_{-h}(R_0,r,t)] + r^hC_{13}(t) + r^{-h}C_{14}(t),\\
\tilde{\sigma}_r^{(1)}(r,t) = (2hgr)^{-1}\{q_1[(h-1)M_h(R_0,r,t) - hN_h(R_0,r,t)]\\
- q_2[(h+1)M_{-h}(R_0,r,t) - hN_{-h}(R_0,r,t)]\} + q_1r^{h-1}C_{13}(t) - q_2r^{-h-1}C_{14}(t),\\
\tilde{\sigma}_\varphi^{(1)}(r,t) = (2hgr)^{-1}\{s_1[(h-1)M_h(R_0,r,t) - hN_h(R_0,r,t)]\\
- s_2[(h+1)M_{-h}(R_0,r,t) - hN_{-h}(R_0,r,t)]\} + s_1r^{h-1}C_{13}(t) - s_2r^{-h-1}C_{14}(t) - g^{-1}[\lambda k_1(r,t) + \mu m\theta_1(r,t)],\\
\tilde{\sigma}_z^{(1)}(r,t) = (2hgr)^{-1}\{q_1[(h-1)M_h(R_0,r,t) - hN_h(R_0,r,t)]\\
- q_2[(h+1)M_{-h}(R_0,r,t) - hN_{-h}(R_0,r,t)]\} + q_1r^{h-1}C_{13}(t) - q_2r^{-h-1}C_{14}(t) - 2k_1(r,t),\\
p_r^{(1)}(r,t) = (4\mu gr)^{-1}\{\mu[hN_{-h}(r,t) - (h+1)M_{-h}(r,t) - 2gC_{14}(t)]\\
+ \mu[(h-1)M_h(r,t) - hN_h(r,t) - 2ghC_{13}(t)] - (2g+\mu)r[k_1(r,t) - \mu m\theta_1(r,t)]\},\\
N_h(r_0,r,t) = r^h\int_{r_0}^{r} r^{-h}k_1(\rho,t)\,d\rho, \qquad M_h(r_0,r,t) = mr^h\int_{r_0}^{r} r^{-h}\theta_1(\rho,t)\,d\rho,\\
h = \sqrt{wg^{-1}}, \quad q_1 = hg + \lambda, \quad q_2 = hg - \lambda, \quad s_1 = h\lambda + w, \quad s_2 = h\lambda - w.
\end{gathered}
\tag{15}
$$

Due to the presence of the new flow region, it is necessary to determine all unknown functions of integration defined for the regions of thermoelastic deformation $m(t) \leqslant r \leqslant R_1$ and $n(t) \leqslant r \leqslant R_2$ in (11) for the flow region $R_1 \leqslant r \leqslant n(t)$ in (14), and for the flow region $R_0 \leqslant r \leqslant m(t)$ in (15). It is also required to re-define the function $n(t)$ ($t > t_2$) and $m(t)$.

The constructed solution, which takes into account the presence of two regions of plastic flow, depends on time. Temperature equalization in the materials of the assembly elements and their cooling lead to flow deceleration and unloading. Calculations have shown that at the time $t = t_3$ ($t_3 > t_2$) in the neighborhood of the contact surface $r = R_1$, a new boundary between the regions of elastic and plastic flow $r = h(t)$ appears such that, in the region $R_1 \leqslant r \leqslant h(t)$, the material continues to deform elastically but there are accumulated irreversible strains. Such plastic strains does not change, but they need to be taken into account in the equilibrium equations written in displacements:

$$\approx{u}^{(2)}_{r,rr} + (r^{-1}\approx{u}^{(2)}_{r})_{,r} - 2\mu w^{-1}(p^{(2)}_{r,r} + 2r^{-1}p^{(2)}_{r}) = mw^{-1}(\theta_2)_{,r}. \quad (16)$$

Here the double tilde above the stress and displacement components indicates that they correspond to the unloading region.

The irreversible (plastic) strains $p^{(2)}_r$ in (16) are functions of only the spatial coordinate r. Integrating (16), we obtain the displacements $\approx{u}^{(2)}_r(r,t)$ in the region $R_1 \leqslant r \leqslant h(t)$; the obtained displacements and temperature are used to calculate the stresses $\approx{\sigma}^{(2)}_r(r,t)$ and $\approx{\sigma}^{(2)}_\varphi(r,t)$ in this region of deformation:

$$\approx{u}^{(2)}_r(r,t) = rw^{-1}F_2(R_1,r,t) + 2\mu rw^{-1}P_2(r) + rC_{25}(t) + r^{-1}C_{26}(t),$$

$$\approx{\sigma}^{(2)}_r(r,t) = -2\mu w^{-1}F_2(R_1,r,t) + 4\mu gw^{-1}P_2(r) + 2gC_{25}(t) - 2\mu r^{-2}C_{26}(t),$$

$$\approx{\sigma}^{(2)}_\varphi(r,t) = 2\mu w^{-1}(F_2(R_1,r,t) - m\theta_2(r,t)) + 4\mu gw^{-1}(P_2(r) + p^{(2)}_r(r)) + 2gC_{25}(t) + 2\mu r^{-2}C_{26}(t),$$

$$\approx{\sigma}^{(2)}_z(r,t) = 2\mu\lambda w^{-1}(p^{(2)}_r(r) + 2P_2(r) - 2m\lambda^{-1}\theta_2(r,t)) + 2\lambda C_{25}(t),$$

$$P_v(r) = \int_{R_{v-1}}^{r} \rho^{-1}p^{(v)}_r(\rho)\,d\rho.$$

The unknown functions of time $C_{25}(t)$ and $C_{26}(t)$, which arise from the integration of (16) need to be determined from the boundary conditions. Other similar functions in Eqs. (11), (14), and (15) must also be redefined. The existing boundary conditions on the boundary surfaces $r = R_0$ and $r = R_2$ and on the contact surface $r = R_1$ are supplemented by the conditions on the boundaries of the regions of elastic and plastic flows $r = n(t)$ and $r = m(t)$. This system also includes two new relations on the boundary between the regions of plasticity and elastic unloading $r = h(t)$: $\approx{\sigma}^{(2)}_r(h,t) = \tilde{\sigma}^{(2)}_r(h,t)$ and $\approx{u}^{(2)}_r(h,t) = \tilde{u}^{(2)}_r(h,t)$. The functions of time obtained in this manner are lengthy and are not presented here.

Deceleration of plastic flow in the inner tube material begins at the time $t = t_4$ ($t_4 > t_3$). At this time, the boundary between the regions of plasticity and elastic unloading $r = q(t)$ appears in the vicinity of the free surface $r = R_0$. In the region $R_0 \leqslant r \leqslant q(t)$, the deformation law is similar to (16). The equilibrium equation in displacements it is written as

$$\approx{u}^{(1)}_{r,rr} + (r^{-1}\approx{u}^{(1)}_{r})_{,r} - 2\mu w^{-1}(p^{(1)}_{r,r} + r^{-1}p^{(1)}_{r}) = mw^{-1}(\theta_1)_{,r}. \quad (17)$$

Integration of (17) yields

$$\approx{u}^{(1)}_r(r,t) = rw^{-1}F_1(R_0,r,t) + \mu w^{-1}(rP_1(r) + r^{-1}Q_1(r)) + rC_{15}(t) + r^{-1}C_{16}(t),$$

$$\approx{\sigma}^{(1)}_r(r,t) = -2\mu w^{-1}F_1(R_0,r,t) + 2\mu w^{-1}(gP_1(r) - \mu r^{-2}Q_1(r)) + 2gC_{25}(t) - 2\mu r^{-2}C_{26}(t),$$

$$\approx{\sigma}^{(1)}_\varphi(r,t) = 2\mu w^{-1}(F_1(R_0,r,t) - m\theta_1(r,t)) + 2\lambda\mu w^{-1}p^{(1)}_r(r) + 2\mu w^{-1}(gP_1(r) + \mu r^{-2}Q_1(r)) + 2gC_{15}(t) + 2\mu r^{-2}C_{16}(t),$$

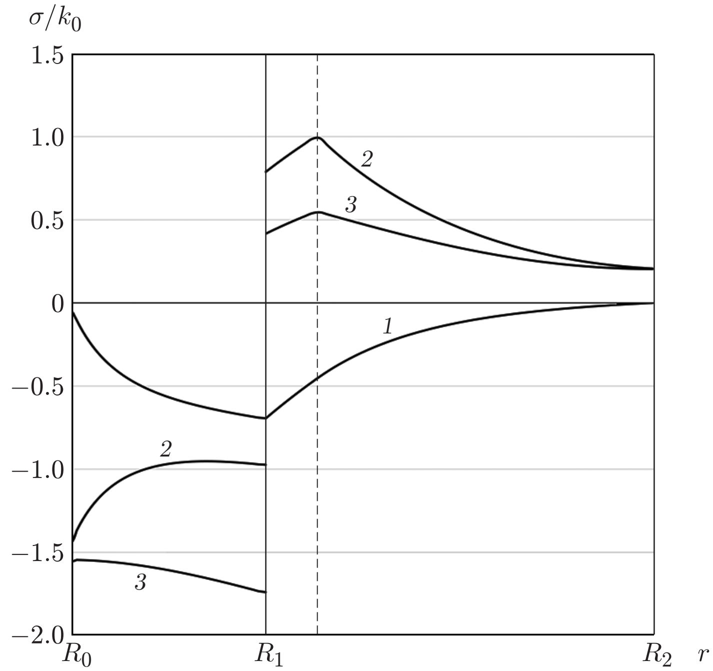

Fig. 1. Stress distribution in the materials of cylindrical bodies at the time of onset of plastic flow in the inner cylinder: σ_r *(1)*, σ_φ *(2)*, and σ_z *(3)*; the vertical lines are the boundaries of the regions of plastic flow.

$$\tilde{\tilde{\sigma}}_z^{(1)}(r,t) = 2\mu w^{-1}(\lambda P_1(r) + g p_r^{(1)}(r)) + 2\lambda C_{15}(t) + 2mgw^{-1}\theta_1(r,t),$$

$$Q_v(r) = \int\limits_{R_{v-1}}^{r} p_r^{(v)}(\rho)\rho\,d\rho.$$

Supplementing the solutions formulated at the previous stages by the boundary conditions on the moving boundary between the regions of elastic deformation and plastic flow $r = q(t)$ [$\tilde{\sigma}_r^{(1)}(q,t) = \tilde{\tilde{\sigma}}_r^{(1)}(q,t)$ and $\tilde{u}_1(q,t) = \tilde{\tilde{u}}_1(q,t)$], we obtain all the unknown functions of time, including $n(t)$, $m(t)$, $h(t)$, and $q(t)$. In determining $n(t)$ and $m(t)$, we use the condition of zero plastic strains, and in determining $h(t)$ and $q(t)$, the condition of zero plastic strain rates.

Over time, the boundary of $r = h(t)$ first catches up with the surface $r = n(t)$, and then the boundary $r = q(t)$ coincides with the surface $r = m(t)$. Thus, the flow regions disappear, and only the regions with accumulated plastic strain remain. Further the temperature in the material of the assembly elements is equalized. At the time when the temperature becomes independent of the spatial coordinate r, the stresses across the assembly stop changing until it is completely cooled.

Figures 1 and 2 show typical stress distribution over the elements of the assembly versus the spatial coordinate r. The stress distribution at the time preceding the time of the onset of plastic flow is shown in Fig. 1, and the distribution of residual stresses in Fig. 2. The assembly material is assumed to be St. 45 steel, and the yield stress at room temperature is 360 MPa. Note that on the contact surface $r = R_1$, there is a discontinuity of the stress σ_φ. The presence of plastic flow leads to a reduction in the final contact stress σ_r on the surface $r = R_1$ compared with the calculated values obtained using the theory of elastic thermal stresses (see Fig. 2).

In conclusion, we note the following feature of quasistatic unloading processes due to thermal stresses. In most of the well-known exact solutions of isothermal problems of the development and deceleration of plastic flows [13–15], the boundary between the regions of elastic deformation and plastic flow stops during unloading, and a new boundary begins to move from it in the opposite direction, behind which flow is absent. In the present case, where only thermal action is considered, the boundaries of the regions of plasticity and elastic unloading always

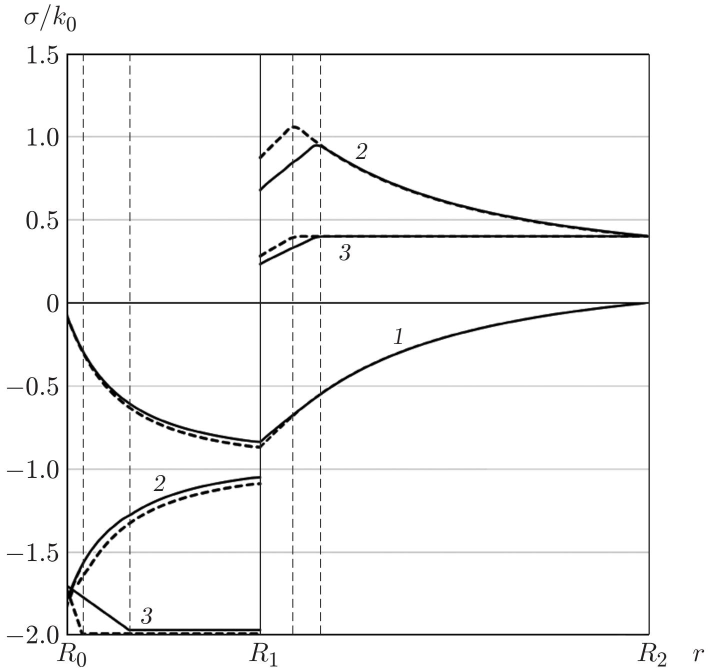

Fig. 2. Distribution of residual stresses in the assembly material: σ_r (*1*), σ_φ (*2*), and σ_z (*3*); the dashed curves are the stresses in the case of a linear temperature dependence of the yield stress; the vertical lines are the boundaries of the regions of plastic flow.

move from the boundary surfaces $r = R_1$, $r = R_0$, which must be taken into account in formulating problems of thermal stresses and developing appropriate computation algorithms.

The calculations carried out for the case of a linear temperature dependence of the yield stress have shown that the flow region is smaller, the final tension is slightly increased, and the difference in circumferential stress on the contact surface is significantly increased.

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