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On a Method of Temperature Stresses Computation in a Functionally Graded Elastoplastic Material

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Abstract—The paper considers a sequence of solutions to the one-dimensional problem of irreversible deformation of a functionally graded material under conditions of uneven thermal expansion. Numerical solutions are obtained for the problems of heating an elastoplastic sphere, the material constants of which are linear functions of the radius, and exact solutions, in which the material constants are approximated by piecewise constant functions. It is shown that the deformation of a functionally graded elastoplastic material, in which the material constants are specified by piecewise-constant distributions, can be qualitatively described by numerical solutions, in which the material constants are continuous approximations of the corresponding piecewise-constant functions. The obtained numerical and analytical solutions of boundary value problems are graphically analyzed.

1 *Keywords*: plasticity, thermoelasticity, residual stress, temperature stress, deformation

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INTRODUCTION

In modern technological practice, materials with non-uniformly distributed mechanical characteristics are increasingly used [1–4]. Such materials, in particular, include functionally graded materials. Functionally graded materials are, first of all, alloys consisting of hard granules and microinhomogene-
2 ities (carbides, nitrides and borides of transition metals (tungsten carbide, titanium carbide, titanium carbonitride, titanium diboride, etc.). Such inhomogeneities form a solid continuous frame. in this case, additives of cobalt, nickel, titanium, aluminum act as a binder). Its content is continuously changing in the volume of the material, which is easily set by the parameters of additive manufacturing processes (first of all). As a result, functionally graded materials have high hardness and high toughness. Due to this, functionally graded materials are used in the production of military equipment, aviation, in the mining industry and medicine.

Structures assembled from functionally graded materials and alloys are used in a wide temperature range. Therefore, one of the main urgent problems in calculating the stress-strain state and residual stresses in such products and structures is taking into account the thermal expansion of the material. Under conditions of temperature exposure, different coefficients of thermal expansion in a multilayer structure are the cause of the appearance of gradients of thermal stresses and deformations even under conditions of uniform heating of the product. High stress levels can lead to the development of areas of irreversible deformation and destruction.

Theoretical studies of the stress-strain state of an elastoplastic material subjected to intense thermal action are presented in exact solutions of a number of problems within the framework of the theory of thermal stresses [5] and the theory of plastic flow [6]. In [7–9], numerical-analytical one-dimensional solutions for the stress-strain state of a material in an elastoplastic ball were obtained taking into account the temperature dependence of the yield stress. The formation of temperature stresses during plastic flow and unloading under conditions of an axisymmetric temperature distribution was studied in [10–13]. In

[14, 15], a comparison was made of solutions for temperature stresses in an elastoplastic cylinder obtained for three different yield criteria. The work [16] shows the possibility of obtaining analytical solutions for the stress-strain state of an elastoplastic material under conditions of toroidal symmetry. In [17, 18], exact solutions were obtained for the problems of determining residual stresses in a two-layer elastoplastic material of spherical and cylindrical shape at different levels of thermal expansion in each layer.

In the presented work, numerical and analytical solutions of thermoelastoplasticity problems in a spherical multilayer material with different mechanical parameters of the material for each layer are considered. A new approach is proposed to determine the stress-strain state of a functionally graded elastoplastic material using linear approximations of the material parameters. The process of obtaining analytical solutions to boundary value problems of the theory of thermoelastoplasticity is similar to the method described by D.D. Ivlev in [6] for elastoplastic problems.

1. GOVERNING EQUATIONS

Further considerations are carried out within the framework of the classical model of small elastic-plastic deformations of the Prandtl-Reuss type [5], in which the total strain tensor ε_{in} consists of thermoelastic (reversible) e_{ye} and plastic (irreversible) p_{it} parts:

$$\varepsilon_{ij} = e_{ij} + p_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j), \tag{1.1}$$

where u_i are the components of the displacement vector in the Cartesian coordinate system, ∂_j is the operator of partial differentiation with respect to the corresponding spatial coordinate.

The relationship between the components of the tensor of thermal stresses and thermoelastic deformations is determined by the Duhamel-Neumann law:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - (3\lambda + 2\mu)\alpha\Delta)\delta_{ij}, \tag{1.2}$$

where μ and λ are Lame parameters, α is the coefficient of linear thermal expansion, $\Delta = T - T_0$ is the difference between the current and initial temperatures.

Within the framework of the model of elastoplastic deformations, a process of irreversible deformation is possible, the beginning of which is associated with the fulfillment of the yield criterion in some regions

$$f(\sigma_{ij}) = 0. \tag{1.3}$$

According to the principle of maximum dissipation of the Mises energy, the surface determined by the implicit dependence (1.3) turns out to be a plastic potential, from which the relations of the associated law of plastic flow follow:

$$\partial_t \varepsilon_{ij} = \xi \frac{\partial f}{\partial \sigma_{ij}}, \quad \xi = \sqrt{\partial_t \varepsilon_{kl} \partial_t \varepsilon_{lk}} \left(\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{nm}} \right)^{-\frac{1}{2}}, \quad \partial_t = \frac{\partial}{\partial t}. \tag{1.4}$$

Under conditions of a slow change in temperature in a spherical layer, the equilibrium equation is valid, which in the absence of mass forces can be taken in the form

$$\partial_j \sigma_{ji} = 0. \tag{1.5}$$

The system of equations (1.1)–(1.5), taking into account the boundary conditions and the heat conduction equation, completely describes the stress-strain state of a thermoelastoplastic material.

2. STATEMENT AND PROBLEM SOLUTION OF A MULTILAYER STRUCTURE DEFORMATION UNDER SPHERICAL SYMMETRY CONDITIONS

In order to obtain the simplest analytical dependences, the problem statement is implemented in a one-dimensional version. A composite spherical layer is investigated in which the stress-strain state at each point of the material depends only on the radial coordinate and the degree of uniform heating. A functionally graded spherical material is a multilayer structure consisting of n layers of a given thickness, in each of which the material parameters are constant, and their values change when passing from layer to layer. Let us write down the basic relations of theory (1.1)–(1.5) for the layer number v $(1 \le v \le n)$ under the conditions of spherical symmetry:

$$\varepsilon_{rr}^{(v)} = e_{rr}^{(v)} + p_{rr}^{(v)} = \frac{\partial u_r^{(v)}}{\partial r}, \quad \varepsilon_{\theta\theta}^{(v)} = \varepsilon_{\varphi\varphi}^{(v)} = e_{\theta\theta}^{(v)} + p_{\theta\theta}^{(v)} = \frac{u_r^{(v)}}{r},$$
$$\sigma_{rr}^{(v)} = (\lambda_v + 2\mu_v)e_{rr}^{(v)} + 2\lambda_v e_{\theta\theta}^{(v)} - (3\lambda_v + \mu_v)\alpha_v\Delta, \tag{2.1}$$
$$\sigma_{\theta\theta}^{(v)} = 2(\lambda_v + \mu_v)e_{\theta\theta}^{(v)} + \lambda_v e_{rr}^{(v)} - (3\lambda_v + \mu_v)\alpha_v\Delta,$$

By integrating the system of equations (2.1), we obtain the general solution under the conditions of thermoelastic equilibrium

$$\sigma_{rr}^{v} = A_v + \frac{1}{B_v^3}, \quad \sigma_{\theta\theta}^{v} = \sigma_{\varphi\varphi}^{(v)} = A_v - \frac{1}{2B_v^3},$$
$$u_r^{(v)} = r\alpha_v\Delta + \frac{A_v r}{3\lambda_v + 2\mu_v} - \frac{B_v}{4\mu_v r^2}. \tag{2.2}$$

Here A_v, B_v are the unknown constants of integration determined from the boundary conditions of the problem.

The layer size v is determined by the inner and outer radii $R_{v-1} \le r \le R_v$. Thus, the layers are numbered in the direction of increasing the radial coordinate. To find the constants of integration in (2.2), we set the boundary conditions of the problem. In a multilayer material, the conditions for the continuity of radial displacements and stresses on the contact surfaces can be taken in the form:

$$\sigma_{rr}^{(v)}(R_v) = \sigma_{rr}^{(v+1)}(R_v), \quad u_r^{(v)}(R_v) = u_r^{(v+1)}(R_v), \quad 1 \le v \le n-1. \tag{2.3}$$

On the inner surface R_0 of the first layer and on the outer surface R_n of the last layer, the conditions of free thermal expansion are set

$$\sigma_{rr}^{(1)}(R_0) = 0, \quad \sigma_{rr}^{(n)}(R_n) = 0 \tag{2.4}$$

It is assumed that the multilayer material is subject to a uniform increase in temperature. Under conditions of free thermal expansion, the only reason for the occurrence of temperature stresses is the different level of thermal expansion of different layers, caused by different values of the coefficients of linear thermal expansion α_v. The distribution of temperature stresses at different thermal expansion is also influenced by the distribution of the values of the Lame parameters and the size of each layer. Further, in the calculations, a three-layer material with the same thickness for each layer will be used, and the distributions of the material constants will be piecewise constant functions of the radius with the same difference between the values during the sequential transition from layer to layer. Obviously, in the thermoelastic case, solutions with piecewise constant distributions of material constants can be approximated in an elementary way by solutions in which the material constants are some continuous distributions. Of much greater interest is the case of nonlinear behavior of a functionally graded material. In particular, different thermal expansion of the layers can lead to such levels of thermal stresses at which the process of plastic flow can be observed on the contact surfaces. As noted earlier, the onset of plastic flow is determined using the yield criterion (1.3), which in the case of spherical symmetry has the form

$$f(\sigma_{ij}^{(v)}) = (\sigma_{rr}^{(v)} - \sigma_{\theta\theta}^{(v)})^2 - 4k_v^2 = 0, \tag{2.5}$$

where k_v is the pure shear yield strength of the material.

From the associated law of plastic flow under condition (2.5), the relations for the plastic incompressibility of the material follow:

$$p_{rr}^{(v)} + p_{\theta\theta}^{(v)} + p_{\varphi\varphi}^{(v)} = 0, \quad p_{\theta\theta}^{(v)} = p_{\varphi\varphi}^{(v)}. \tag{2.6}$$

Having integrated the equilibrium equations (2.1) taking into account relations (2.5)–(2.6), we obtain solutions for stresses and displacements in the regions of irreversible deformation $R_{v-1} \le r \le a_v$, where a_v is the elastoplastic boundary:

$$\sigma_{rr}^{(v)} = F_v - 4s_v k_v \ln(r), \quad \sigma_{\theta\theta}^{(v)} = \sigma_{\varphi\varphi}^{(v)} = F_v - 4s_v k_v \ln(r) - 2s_v k_v,$$
$$u_r^{(v)} = \frac{G_v}{r^2} - \frac{4s_v k_v}{3\lambda_v + 2\mu_v} r \ln(r) + r\left(\frac{F_v}{3\lambda_v + 2\mu_v} + \alpha_v \Delta\right), \tag{2.7}$$
$$s_v = sign(\sigma_{rr}^{(v)}(a_v) - \sigma_{\theta\theta}^{(v)}(a_v)).$$

To determine the constants F_v, G_v and the constants from (2.2), it is necessary to add to the boundary conditions (2.4) and the conditions on the contact surfaces (2.3) the conditions for the continuity of radial stresses and displacements at the elastoplastic boundaries a_v:

$$\overset{-}{\sigma}_{rr}(a_v) = \overset{+}{\sigma}_{rr}(a_v), \quad \overset{-}{u}_r(a_v) = \overset{+}{u}_r(a_v), \quad v = 0\ldots m \tag{2.8}$$

where m is the number of layers in which the process of irreversible deformation occurs, the minus on top of the symbol determines the value to the left of the elastoplastic boundary, and the plus on top is the value
3 on the right. The positions of the elastoplastic boundaries qi are found by numerically solving a system of equations defining the continuity of circumferential stresses on them:

$$\overset{-}{\sigma}_{\theta\theta}(a_v) = \overset{+}{\sigma}_{\theta\theta}(a_v), \quad v = 0\ldots m \tag{2.9}$$

Thus, to determine the integration constants A_v, B_v, F_v, G_v in an n-layer material with m zones of plastic flow, it is first necessary to solve the system of $2(n+m)$ equations (2.3), (2.4), (2.8), and then, using the found expressions, solve system of m equations (2.9) for determining the position of elastoplastic boundaries a_v.

3. CONTINUOUS DISTRIBUTIONS OF MATERIAL CONSTANTS AND NUMERICAL SOLUTION EXPIREMENTS

Calculation of the stress-strain state of a multilayer material is a difficult task from the point of view of numerical implementation. It is known that solutions of elastoplastic problems in which contact boundary conditions take place require significant computing resources for their convergence. The standard
4 Schwarz algorithm, taking into account the plastic properties of the material, can lead to high errors and, as a consequence, to incorrect solutions for stresses in the contact area. Most often, to simplify the calculation procedure, one would resort to a method for describing a functionally graded material, which consists in representing a multilayer structure in the form of a single continuous layer, in which a discrete distribution of material constants is specified, corresponding to various materials of a given thickness. When investigating within the framework of linear models, in particular under conditions of thermoelastic deformation, this approach allows one to obtain solutions without complicating the calculation procedure and without leading to significant errors in the calculations. However, in the case of taking into account nonlinear effects, such as plastic flow in the vicinity of the points of discontinuity of the distributions of the material constants, this approach leads to computational difficulties associated with correcting solutions when several zones of irreversible deformation on contact surfaces develop. The following is an approach to calculating the temperature stress fields, which consists in representing the stepwise distributions of material constants in a continuous layer using piecewise linear approximations.

The question arises of how to approximate the distributions of the Lame parameters and the coefficient of linear expansion in order to obtain solutions for temperature stresses that describe the stress-strain state of a functionally graded material (which are least different from exact solutions).

Let us consider a three-layer spherical material ($i = 1..3$). Let the material constants have different values in each layer. For convenience, let's assume that each layer has the same thickness, and each material constant can change by the same value as we move from layer to layer. The following are used as referential values for material constants:

$$\lambda_0 = 91\times10^9 \text{ Pa}, \quad \mu_0 = 42\times10^9 \text{ Pa}, \quad \alpha_0 = 17\times10^{-6} \text{ K}^{-1}$$
$$R_0 = 0.2 \text{ m}, \quad T_0 = 300 \text{ K}, \quad k_0 = 300\times10^6 \text{ Pa} \tag{3.1}$$

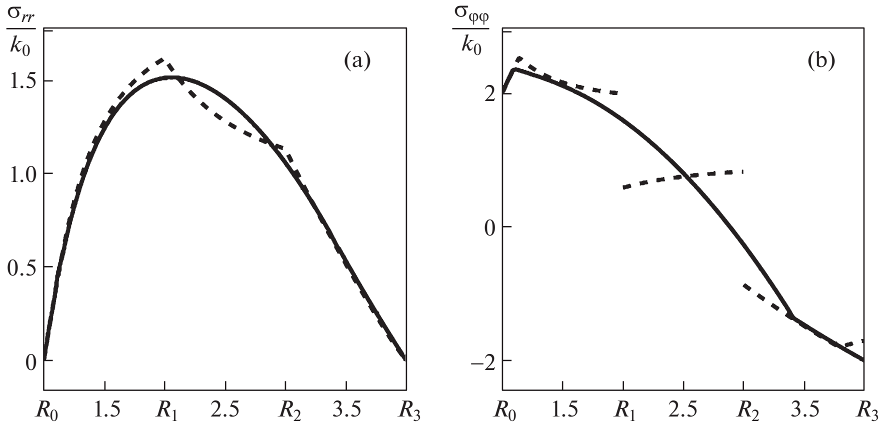

Fig. 1.

In order to analyze the two maximally different stress-strain states of the material, we will take for the material constants in the i-th layer two different ways of specifying their values depending on the layer number. The first method determines the increase in material constants with increasing layer number:

$$R_i/R_0 = i+1, \quad \mu_i/\mu_0 = \lambda_i/\lambda_0 = i, \quad \alpha_i/\alpha_0 = i, \quad k_i = k_0 \tag{3.2}$$

The second method determines the decrease in the values of the material constants as the layer number increases:

$$\mu_i/\mu_0 = \lambda_i/\lambda_0 = 4-i, \quad \alpha_i/\alpha_0 = 4-i, \quad k_i = k_0 \tag{3.3}$$

When constructing continuous functions for the material constants, a linear approximation is used, passing through the values of constants (3.2), (3.3) belonging to the free surfaces of the spherical layer R_0, R_n. As an example, let us write a linear approximation of the coefficient of linear thermal expansion.

$$\overset{q}{\alpha} = (1+w_q)\alpha_n \frac{r-R_0}{R_n-R_0} + (1-w_q)\alpha_1 \frac{r-R_n}{R_0-R_n} \tag{3.4}$$

Hereafter, the index q over the corresponding function denotes the method of approximating the material parameters, which is used in calculating this function. w_q is the deviation of the values of the approximations of the material constants from the corresponding discrete analogs (3.2), (3.3), given on the free surfaces of the spherical layer. The values of w_q and the methods of specifying the constants (3.2), (3.3) determine the differences in approximations (3.4). For material constants (3.2), the parameters in equation (3.4) are written in the form

$$\begin{aligned} &q=1, \quad w_1 = 0, \quad \alpha_1 = \alpha_0, \quad \alpha_n = n\alpha_0 \\ &q=2, \quad w_2 = 0.5, \quad \alpha_1 = \alpha_0, \quad \alpha_n = n\alpha_0 \\ &q=3, \quad w_3 = 0.25, \quad \alpha_1 = \alpha_0, \quad \alpha_n = n\alpha_0 \end{aligned} \tag{3.5}$$

For material constants (3.3), the parameters in equation (3.4) take the form:

$$\begin{aligned} &q=4: \quad w_4 = 0, \quad \alpha_1 = n\alpha_0, \quad \alpha_n = \alpha_0 \\ &q=5: \quad w_5 = 0.5, \quad \alpha_1 = n\alpha_0, \quad \alpha_n = \alpha_0 \\ &q=6: \quad w_6 = 0.25, \quad \alpha_1 = n\alpha_0, \quad \alpha_n = \alpha_0 \end{aligned} \tag{3.6}$$

4. OPTIMAL CALCULATIONS

This section presents the results of calculations under the condition of a continuous distribution of material parameters and a comparison with the results of calculations performed within the approach with

Table 1.

	$\alpha_1 < \alpha_2 < \alpha_3$			$\alpha_1 > \alpha_2 > \alpha_3$		
	$q = 1$	$q = 2$	$q = 3$	$q = 4$	$q = 5$	$q = 6$
$I(\overset{q}{\sigma}_{rr})$	0.301	0.132	0.194	0.482	0.212	0.120
$I(\overset{q}{\sigma}_{\varphi\varphi})$	1.126	0.8223	0.808	1.539	1.238	1.061

a layered material representation. The following condition was chosen as a criterion for the best fit of numerical solutions for stresses to exact solutions:

$$I(\overset{q}{\sigma}_{ij}) = \sum_{v=1}^{3} \int_{R_{v-1}}^{R_v} |\sigma_{ij}^{(v)} - \overset{q}{\sigma}_{ij}|dr \to \min, \tag{4.1}$$

where $\overset{q}{\sigma}_{ij}$, the numerical solutions obtained in the framework of the linear approximation q, determine the number of the approximation (3.5) - (3.6). Numerical modeling was carried out in the Comsol Multiphysics environment. Table 1 shows the values of functional (4.1) for various approximations of the material constants:

5. DISCUSSION OF THE NUMERICAL RESULTS

When performing calculations within the framework of thermoelastic equilibrium, it was found that the best approximation of the constants for the two considered functionally graded materials is achieved at $q = 2$ and $q = 5$, respectively. With this choice of parameters describing the continuous distribution of constants, the smallest value of integral (4.1) is attained. In the case of taking into account nonlinear effects, such as plastic flow on the interlayer contact surfaces, the choice of the approximation depends on the distribution patterns of the material constants and the sizes of the regions of irreversible deformation. For a material described by a set of constants (1.2) (an increase in the values of the material constants in the direction of an increase in radius), the best approximation is $q = 2$ when estimating the radial stress σ_{rr} and $q = 3$ for the circumferential stress $\sigma_{\varphi\varphi}$ (. 1). For a material described by a set of constants (1.3), the best approximation of the constants is $q = 6$ (Fig. 2). This approximation makes it possible to obtain the minimum difference between all components of the stress tensor calculated analytically for a multilayer material and the stresses obtained numerically. Note that an increase in temperature and, as a consequence, an increase in the zones of irreversible deformation with the set of constants (1.3) leads to a decrease in the difference between the two considered methods of calculating stresses, and when the set of constants (1.2), on the contrary, to its increase. This is due to the fact that in a multilayer material, plastic flow within the framework of piecewise constant distributions of material parameters always starts at a

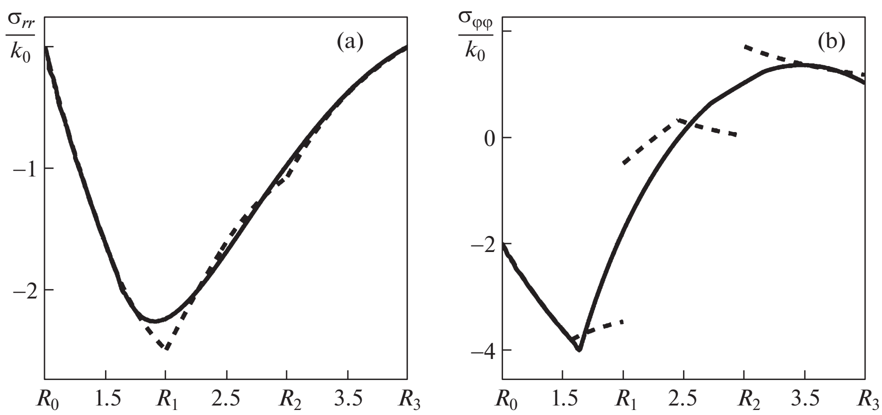

Fig. 2.

smaller radius of the layer and develops in the direction of a larger radius. In this case, the use of continuous distributions of material constants in some cases also leads to the appearance of flows on the inner surface of the spherical layer, and in others, on the contrary, to the appearance of a flow on the outer surface. These features cause a decrease or increase in the difference between stresses as the plastic flows develop. The simplest solution to the problem for three layers makes it possible to indicate the best approximations of the constants depending on the patterns of their distribution in a multilayer material in order to use such approximations to describe functionally graded materials and alloys with a large number of layers.

CONCLUSIONS

The paper considers the sequence of solutions to the one-dimensional problem of irreversible deformation of a functionally graded material under conditions of uneven thermal expansion. Numerical solutions are obtained for the problems of heating an elastoplastic sphere, the material constants of which are linear functions of the radius, and exact solutions, in which the material constants are approximated by step functions. It is shown that the deformation of a functionally graded elastoplastic material, in which the material constants are specified by piecewise-constant distributions, can be qualitatively described by numerical solutions, in which the material constants are continuous approximations of the corresponding step functions. It was found that among the possible linear approximations of the constants there are those at which a minimum difference is achieved between the stress-strain state of a multilayer material and a solid material with a continuous change in its physical characteristics.

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SPELL: 1. thermoelasticity, 2. borides, 3. qi, 4. Schwarz