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Thermal Residual Stresses Computing in Elastic-Plastic Ball with Rigid Inclusion under Heat Treatments

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Abstract—**The present study is devoted to the boundary value problem of cooling of an elastic-plastic additive manufactured spherical ball with rigid inclusion. Throughout the paper the conventional Prandtl–Reuss model of solids generalised on the thermal effects is used. The residual stresses and deformations depending on the cooling rate and the layer size are investigated. An estimate is made of the need to take into account the process of non-stationary heat conductivity in the contact problems of the theory of temperature stresses. Fields of residual stresses and deformations are computed for different cooling modes.**

Index Terms—**elasticity, heat conduction, yield criterion, plasticity, residual strain, thermal stress.**

I. Introduction

THE present study deals with the problem of heat transfer influence on residual stresses formation in elastic-plastic solids.

As it is known, the thermal stresses accumulation occur due to the thermal expansion of the material. However, in a freely expanding solids, a uniform temperature field does not cause stress [1]. In solids having any constraints on thermal expansion (for example, in the case of a fixed part of the body surface, or when an external load is affected) a uniform change in the temperature field can cause significant thermal stresses. Thus, the increasing of stresses with a uniform temperature change depends on the mechanical boundary conditions on the surface of the solids. On the other hand, stresses can also occur in freely expanding bodies under the unevenly distributed temperature field. Different parts of the body are expanded with different rate forming the thermal stresses fields. If we consider the problem of the uneven temperature field effect on a body that has limitations on the thermal expansion. One can conclude that the temperature stresses in the thermoelastic problems arise due to following cases: 1) the stresses arise under the temperature gradient and conditions of free thermal expansion; 2) the stresses develop by the influence of some uniform temperature field $\overline{T}$ (where $\overline{T}$ is the average temperature in the considered body) under limitations of thermal expansion. In the present study we try to estimate the degree of influence of boundary conditions and uneven thermal field on the formation of stresses and deformations in a thermoelastic-plastic material.

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The limitations on thermal expansion include boundary conditions in the contact problems in the frameworks of thermal stresses theory, in which the change in the stress-strain state of the material occurs due to the equalization of the temperature field between the contacting bodies having different referential temperatures. As an example, one can specify shrink fitting processes, in which the stress-strain state is determined by the difference in the initial temperatures in the composite elements of the assembled structures [2]–[4]. Another example is the problem of heating bodies with a rigid core [1]. Uniform temperature field can be replaced by an equivalent mechanical load, when normal stresses are specified on the boundary surfaces, the magnitude of which depends on the temperature level. At present, the additive manufacturing simulations are an exciting area of the modern industry and mechanical engineering [5]–[10]. One way of additive technology processing is adding preheated layers to the basic growing body. The significant extension is formed on the contact surfaces due to the heat transfer determining the strength characteristics of the created objects [4]. The residual stresses in the thermoelastic solids do not depend on the process of heat transfer between the parts and are determined by the steady temperature field. In real materials, such as metals, the occurrence of irreversible deformation processes is possible. The appearance of plastic deformations leads to a change in the residual stress fields. The final distribution of the stress-strain state parameters can be more accurately prescribed by the plastic properties of the material. A special feature of calculating thermal stresses under conditions of irreversible deformation is the possibility of simultaneous occurrence of regions of plastic flow and unloading [11]–[16]. The level of plastic deformation in the material without external loads depends on the magnitude of the temperature field gradient. Regions of plastic flow arise under high values of the temperature gradient. The growth of plastic deformations are terminated and the thermoelastic regions with accumulated irreversible deformations are developed during temperature equalization [13], [15], [17]–[22]. Thus, the equalization of the temperature field in this case leads to the residual stresses formation. It is also possible plastic flow region expansion in the absence of a temperature gradient with particular restrictions on thermal deformation. In this case, the thermoplastic state of the material (the state of neutral loading) is determined by the steady temperature field. Simultaneous considerations of arbitrary boundary conditions and temperature gradient can lead to the repeated appearance, development and disappearance of plastic flow processes [11], [21], [23]–[25]. Similar material behaviour in 2D and 3D problems [26] equires huge computing resources. At the same time, it was found that in some shrink fit processes [3], [27], [28] the distribution of residual stresses approximately coincides with calculations in which the residual stress field is calculated on the basis of a final uniform temperature distribution. In the presented study, the effect of the non-stationary temperature gradient on residual stress formation in an elastic-plastic spherical layers with a central rigid inclusion is considered. The simplest one-dimensional statement of the problem made it possible to elucidate the features of the formation of residual stresses under conditions of non-stationary thermal action and to determine the material parameters (size, thermal diffusivity, initial and final temperature) for the problem reducing by following assumption. We can neglect by the non-stationary thermal conductivity processes. This simplification makes it possible to reduce the solution of the problem by the use of well known methods for cases of isothermal loading of the material.

II. Constitutive equations and boundary value problem statement

Let us consider a heated elastic-plastic spherical layer under initial temperature T_0, with inner and outer radii r_1 and r_2. There is no radial displacement u_r on the inner surface of layer (inclusion surface)

$$u(r_1,t)=0. \tag{1}$$

The condition of free thermal expansion (zero (normal) radial stress) can be furnished on the outer surface in form

$$\sigma_r(r_2,t)=0. \tag{2}$$

It is assumed in the referential state that there is no thermal expansion in the material of the layer

$$\Delta=\alpha(T-T_0), \tag{3}$$

where α is coefficient of linear thermal expansion, T is the actual temperature field.

The resulting thermal stresses σ_i and deformations d_i obey the equations

$$\begin{gathered}\sigma_{r,r}+\frac{\sigma_r-\sigma_\varphi}{r}+\frac{\sigma_r-\sigma_\vartheta}{r}=0,\\ d_{\varphi,r}=\frac{d_r-d_\varphi}{r}.\end{gathered} \tag{4}$$

Hereafter, the index after the comma denotes a partial differentiation with respect to the spatial variable. From the system of equations (4) we can obtain

$$\begin{gathered}\sigma_\varphi=\sigma_\vartheta=\frac{1}{2r}(r^2\sigma_{r,r})_{,r}\\ d_r=(rd_{\varphi,r})_{,r},\quad d_\varphi=d_\vartheta.\end{gathered} \tag{5}$$

Further, taking into account (5), we will assume that the stress tensor is known if its radial component σ_r is known, and the strain tensor is known if a radial displacement is known u_r.

The constitutive equations between stresses and strains in elastic-plastic material is given by the Duhamel-Neumann rules

$$\begin{gathered}\sigma_r=(\lambda+2\mu)(u_{r,r}-p_r)+2\lambda(r^{-1}u_r-p_\varphi)-q\Delta,\\ \sigma_\varphi=2(\lambda+\mu)(r^{-1}u_r-p_\varphi)+\lambda(u_{r,r}-p_r)-q\Delta,\end{gathered} \tag{6}$$

wherein p_i are components of plastic deformations, calculated during the process of plastic flow. The following condition $p_i=0$ is valid in the case of thermoelastic deformation prior to the plastic flow.

The plastic flow process is initialized when the plasticity condition is satisfied

$$f(r,t)=\sigma_\varphi-\sigma_r=2k, \tag{7}$$

where k is yield stress of the material under shear. The relations between the increments of plastic deformations according to associated flow rule read by

$$dp_{rr}=-2dp_{\varphi\varphi}=-2dp_{\vartheta\vartheta} \tag{8}$$

System of the equations (4) – (8) describes the processes of elastic-plastic deformation as a function of the temperature field.

Suppose that at some time $t=t_0$ on the inner surface of the layer the temperature decreases according to the rule

$$\frac{T_0-T}{T_0-T_{min}}=W(t), \tag{9}$$

where T_{min} is the temperature of the cooled layer, x is parameter determining the rate of cooling of the internal surface. The law of temperature change (9) has the form

$$W(t)=(1-\exp(-xt)),$$

which corresponds to the process of rapid cooling of the surface during heat transfer [21]. The outer surface of the layer is insulated from heat loss

$$T_{,r}(r_2)=0.$$

The temperature field is known at any time $t>t_0$ as the numerical solution of the heat conduction equation

$$T_{,t}=r^{-1}(r(T_{,r}))_{,r}.$$

III. Solution of the problem

The differential equation for determining of the radial component of the stress tensor can be derived from the system of equations (4) – (6) as follows

$$(r\sigma_{r,r})_{,r}+3\sigma_{r,r}=-4\omega\Delta_{,r}, \tag{10}$$

where $\omega=\mu(\lambda+2\mu)/q$.

Functions of radial stress and displacement can be obtained by integration of the equation (10)

$$\begin{gathered}\sigma_r=-\frac{4\omega}{r^3}\int_{r_1}^{r}\Delta(\rho,t)\rho^2d\rho+A(t)+\frac{B(t)}{r^3},\\ u_r=\frac{\omega}{\mu r^2}\int_{r_1}^{r}\Delta(\rho,t)\rho^2d\rho+\frac{A(t)}{q}-\frac{B(t)}{4\mu r^2}.\end{gathered} \tag{11}$$

Here $A(t)$, $B(t)$ is functions determining from the boundary conditions of the problem.

The structure of equation (10) allows us to submit his solution by the sum of two terms

1) the general solution of the homogeneous equation

$$\begin{gathered}(r\overline{\sigma}_{r,r}+3\overline{\sigma}_r)_{,r}=0\\ \overline{\sigma}_r=\overline{A}+\frac{\overline{B}}{r^3},\\ \overline{u}_r=r\alpha\overline{T}+\frac{r\overline{A}}{q}-\frac{\overline{B}}{4\mu r^2},\end{gathered} \tag{12}$$

2) particular solution σ_r^* of the inhomogeneous equation (10) under homogeneous boundary conditions $\sigma_r^*(r_i, t) = 0$.

Temperature $\overline{T}$ is the average value of the thermal distribution over the volume of the spherical layer

$$\overline{T}(t) = \frac{3}{r_2^3 - r_1^3} \int_{r_1}^{r_2} \Delta(\rho, t)\rho^2 d\rho. \qquad (13)$$

The plasticity function is represented in the form

$$f(\sigma_i) = f(\sigma_i^*) + f(\overline{\sigma_i}).$$

Suppose that at time $t = t^*$ the following condition

$$f_{,t}(\sigma_i^*) = 0$$

is valid. This equality with free thermal expansion means the unloading of the material. the beginning of unloading depends on the rate of equalization of the thermal field. It is established that, within the framework of the boundary conditions (1)-(2) on the inner surface at $t > t^*$ the inequality

$$f_{,t}(\sigma_i) > 0$$

is satisfied under the monotonous change of temperature field. Thus, the decrease $\overline{T}$ is a sufficient condition for the implementation of the simple loading regime. In this case, the final distribution of stresses under plastic flow does not depend on the thermal conductivity process.

Equivalent definition of simple loading is a constant increase of the level of plastic deformations. In accordance with the condition (7) the circumferential components of the plastic deformations take the form

$$p_\varphi = p_\vartheta = \frac{k}{\omega} + \frac{3}{r^3} \int_{r_1}^{r} \Delta(\rho, t)\rho^2 d\rho - \Delta(r, t). \qquad (14)$$

The boundary of the plasticity region at each time $r = a(t)$ is calculated from condition $p_\varphi(a, t) = 0$.

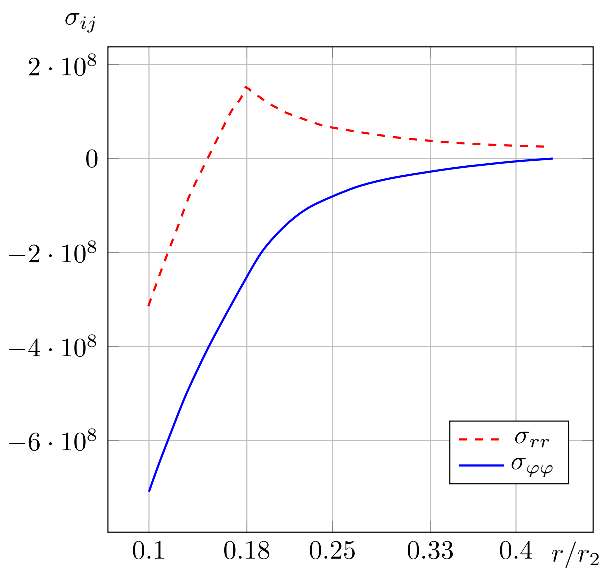

Fig. 1. Residual stresses, $r_2/r_1 = 4.3$.

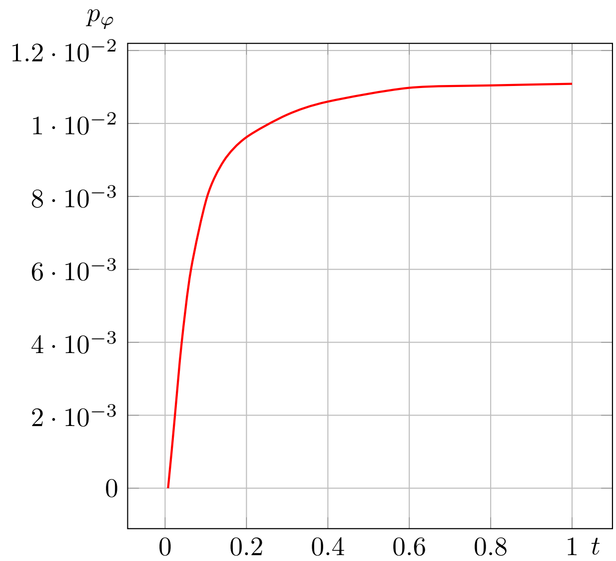

Fig. 2. The evolution of plastic deformation in time on the inner surface. $T(r_1, t) = W_1(t)$, $x = 20$.

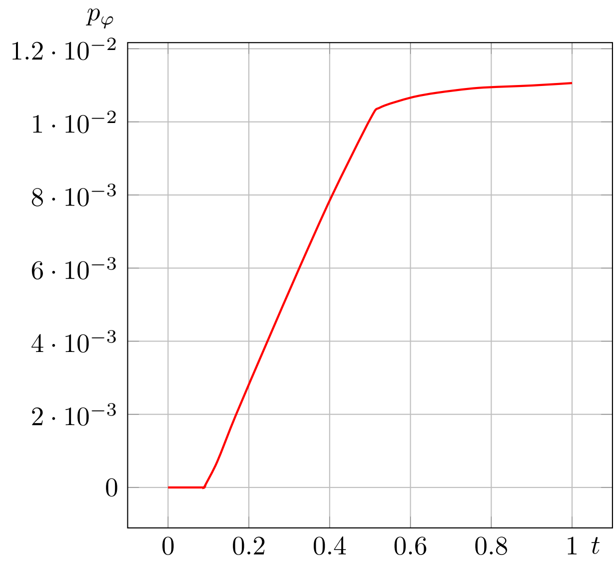

Fig. 3. The evolution of plastic deformation in time on the inner surface. $T(r_1, t) = W_2(t)$, $t_1 = 0.5$.

IV. Concluding Remarks and Results Discussion

The residual stresses are shown on Fig.(1) for an arbitrary law of cooling on the contact surface. The carried out calculations make it possible to conclude a number of important remarks on the possibility of affect of non-stationary thermal conductivity on irreversible deformation processes. The maximum value of the loading function $f(\sigma_i)$ is reached on the surface of contact with a rigid inclusion. This circumstance means that among the entire volume of the spherical layer the maximum change in plastic deformation affecting the formation of residual stresses occurs on the contact surface. On Fig. 2 it is seen that the law of variation of the circumferential plastic deformation with time corresponds to a monotonically increasing function. Thus, the process of irreversible deformation is a simple loading mode, in which the final resulting distribution does not depend on the loading history (hardening or possible appearance of unloading regions). The level of plastic deformations is proportional to the value $T_0(r_2 - r_1)/r_1$ and at the final time does not depend on the cooling speed. Function type (9) determines the behavior of plastic deformation on the inner surface of the layer. In the general case, for an arbitrary non-decreasing function that specifies the temperature boundary condition (9), the decrease of the average temperature and the simple loading regime are valid, at which there is no intermediate unloading of the material, and, consequently, the final elastic-plastic state is completely described by the final thermal field distribution. The change of plastic deformations in time is shown on Fig. 3 under the temperature boundary condition in the form $W_2(t) = (1 - t/t_1)$. Rapid growth of deformations in the range $t_0 < t < t_1$ is due to a high level of temperature gradient. Further changes of deformation occur due to temperature equalization and increase of mechanical stresses. Note also that the cooling rate parameter x (9) does not affect the distribution of residual strains and under conditions of ideal plasticity is insignificant.

References

[1] B. Boley and J. Weiner, *Theory of Thermal Stresses*. New York: Wiley, 1964.
[2] A. A. Burenin, E. P. Dats, and A. V. Tkacheva, "On the modeling of the shrink fit technology," *Journal of Applied and Industrial Mathematics*, vol. 8, no. 4, pp. 493–499, 2014.
[3] E. P. Dats and A. V. Tkacheva, "Technological thermal stresses in the shrink fitting of cylindrical bodies with consideration of plastic flows," *Journal of Applied Mechanics and Technical Physics*, vol. 57, no. 3, pp. 569–576, 2016.
[4] E. V. Murashkin and E. P. Dats, "Thermoelastoplastic deformation of a multilayer ball," *Mechanics of Solids*, vol. 52, no. 5, pp. 30–36, 2017.
[5] A. V. Manzhirov and D. A. Parshin, "Accretion of a viscoelastic ball in a centrally symmetric force field," *Mechanics of Solids*, vol. 41, p. 51–64, 2006.
[6] S. I. Kuznetsov, A. V. Manzhirov, and I. Fedotov, "Heat conduction problem for a growing ball," *Mechanics of Solids*, vol. 46, p. 929–936, 2011.
[7] A. V. Manzhirov and D. A. Parshin, "Arch structure erection by an additive manufacturing technology under the action of the gravity force," *Mechanics of Solids*, vol. 50, p. 559–570, 2015.
[8] A. V. Manzhirov, "Influence of the erection regime on the stress state of a viscoelastic arched structure erected by an additive technology under the force of gravity," *Mechanics of Solids*, vol. 50, p. 657–675, 2015.
[9] A. V. Manzhirov, "Advances in the theory of surface growth with applications to additive manufacturing technologies," *Procedia Engineering*, vol. 173, pp. 11–16, 2017.
[10] A. V. Manzhirov, "Mechanical design of am fabricated prismatic rods under torsion," *MATEC Web of Conferences*, vol. 95, p. 12002, 2017.
[11] E. Dats, S. Mokrin, and E. Murashkin, "Calculation of the residual stresses of hollow cylinder under unsteady thermal action," *Lecture Notes in Engineering and Computer Science*, vol. 2218, pp. 1043–1046, 2015.
[12] E. Dats, S. Mokrin and E. Murashkin, "Calculation of the residual stress field of the thin circular plate under unsteady thermal action," *Key Engineering Materials*, vol. 685, pp. 37–41, 2016.
[13] A. A. Burenin, E. P. Dats, S. Mokrin, and E. V. Murashkin, "Plastic flow and unloading of a hollow cylinder during the "heating-cooling" action," *Bulletin of Yakovlev Chuvash State Pedagogical University. Series: Mechanics of Limit State*, vol. 16, no. 2, pp. 22–28, 2013.
[14] E. P. Dats, E. V. Murashkin, and R. Velmurugan, "Calculation of irreversible deformations in a hollow elastic-plastic sphere under non-steady thermal action," *Bulletin of Yakovlev Chuvash State Pedagogical University. Series: Mechanics of Limit State*, vol. 25, no. 3, pp. 168–175, 2015.
[15] G. Parkus, *Unsteady Thermal Stresses*. Moscow: Fizmatgiz, 1963.
[16] E. Murashkin, E. Dats, and V. Klindukhov, "Numerical analysis of the elastic-plastic boundaries in the thermal stresses theory frameworks," *Journal of Physics: Conference Series*, vol. 937, no. 1, pp. 1–6, 2017.
[17] E. Dats, N. Stadnik, and E. Murashkin, "Piecewise linear yield criteria in the problems of thermoplasticity," *IAENG International Journal of Applied Mathematics*, vol. 47, no. 3, pp. 261–264, 2017.
[18] E. Dats, N. Stadnik, and E. Murashkin, "On heating of thin circular elastic-plastic plate with the yield stress depending on temperature," *Procedia Engineering*, vol. 173, pp. 891–896, 2017.
[19] A. A. Burenin, E. P. Dats, and E. V. Murashkin, "Formation of the residual stress field under local thermal action," *Mechanics of Solids*, vol. 49, no. 2, pp. 218–224, 2014.
[20] B. Raniecki, "Stresses in an elastic-plastic hollow sphere subjected to a variable temperature field," *Rozpr. Inz.*, vol. 14, p. 479, 1966.
[21] H. Odeno, "Transient thermal stresses in elasto-plastic discs," *Journal Mechnical Engineering Science*, vol. 2, pp. 384–391, 1969.
[22] E. Dats and E. Murashkin, "On unsteady heat effect in center of the elastic-plastic disk," *Lecture Notes in Engineering and Computer Science*, vol. 2223, pp. 69–72, 2016.
[23] Y. Orcan, "Residual stresses and secondary plastic flow in a heat generating elastic-plastic cylinder with free ends," *International Journal of Mechanical Sciences*, vol. 33, pp. 1689–1698, 1995.
[24] H. Lippmann, "The effect of a temperature cycle on the stress distribution in a shrink fit," *International Journal of Plasticity*, vol. 8, pp. 567–582, 1992.
[25] M. Bengeri and W. Mack, "The influence of the temperature dependence of the yield stress on the stress distribution in a thermally assembled elastic-plastic shrink fit," *Acta Mechanica*, vol. 103, pp. 243–257, 1994.
[26] S. A. Gorshkov, E. P. Dats, and E. V. Murashkin, "Calculation of the plain field of thermal stresses under plastic flow and unloading," *Bulletin of Yakovlev Chuvash State Pedagogical University. Series: Mechanics of Limit State*, vol. 21, no. 3, pp. 169–175, 2015.
[27] A. Kovacs, "Thermal stresses in a shrink fit due to an inhomogeneous temperature distribution," *Acta Mechanica*, vol. 105, pp. 173–187, 1994.
[28] A. Kovacs, "Residual stresses in thermally loaded shrink fits," *Periodica Polytechnica. Ser.: Mech. Eng.*, vol. 40, pp. 103–112, 1996.