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Calculation of the Residual Stress Field of the Thin Circular Plate under Unsteady Thermal Action

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Abstract. The dimensional problem of a formation of the residual stresses in the thin circular elastoplastic plate under the given thermal action was analytically solved. The generalized Prandtl-Reuss thermoelastoplastic model was used. The effect of the non-stationary temperature gradient on the residual stresses field formation was investigated under the condition that the yield stress depends on a temperature. The borders of the irreversible deformation domain and unloading domain were computed. The level of residual stresses was calculated.

Introduction

The field of residual stresses is formed in the process of elastoplastic deformation with subsequent unloading of the material. It is known that residual stresses may arise because of local thermal actions, for example, near the welding joints [1]. Accounting of such strains and stresses is needed for accurate determination of the geometry and strength characteristics of the concerned materials. Temperature fields have influence on the yield stress of material, by increasing a probability of the non-reversible deformations appearance. Detailed analysis of stress-strain state of an elastoplastic thin plate with infinity size was considered [2]. The features of formation of non-reversible deformation fields were observed for the finite size solid cylindrical body having inner heat source [3]. The numerical solutions of the shrink fit problem for hollow discs were compared using the Mises yield condition and Tresca yield condition [4]. The analytical solution of the shrink fit problem for thin circular plates was considered in condition of yield strength dependence on temperature [5].

This work presents the exact solution of the residual stresses formation problem using the assumption that the connection between the processes of heat conduction and deformation under the conditions of intensive thermal action can be neglected, i.e., the calculations can be performed in the framework of the theory of thermal stresses [6]. The features of residual strains and stresses formation for the load-free thin circular plate with rapidly changeable temperature gradient on the edge were investigated. The method for determining the non-reversible deformations on the boundary between the plastic flow domain and the unloading domain was shown and the residual strains and stresses were calculated.

Mathematical model

Boundary conditions for the plate with a radius R are written in the form:

$$\sigma_{rr}(R,t)=0, \quad u_r(0,t)=0 \tag{1}$$

where u_r is the radial component of the displacement vector, σ_{rr} is the radial component of the stress tensor.

At the initial time $t=0$ the temperature of the plate is $T(r,0)=T_0$. At the time $t>0$ on the edge and center of the plate the following conditions are satisfied:

$$T(R,t) = T_k(1 - \exp(-xt)), \quad T_{,r}(r,t)\big|_{r=0} = 0, \tag{2}$$

where x is the parameter defining the temperature rate increasing on the edge of the plate. The subscript after comma denotes derivation with respect to the corresponding spatial coordinate. At the time $t \to \infty$ the determination of a constant temperature $T(R) = T_k$ follows from the equation (2).

Temperature field is described by the heat equation:

$$T_{,t} = \frac{\chi}{r}\left(rT_{,r}\right)_{,r}, \tag{3}$$

where χ is the thermal diffusivity of a material.

Taking into consideration that infinitesimal strains arising due to the thermal action (3) relations for the radial and angular components of strain have the following form:

$$d_{rr} = u_{r,r} = e_{rr} + p_{rr}, \quad d_{\varphi\varphi} = \frac{u_r}{r} = e_{\varphi\varphi} + p_{\varphi\varphi}, \tag{4}$$

where e_{ij}, p_{ij} are the elastic and plastic components of the strain tensor. Stresses are determined by the thermoelastic strains according to the Duhamel-Neumann Law and could be written for the plane-stress state [2]:

$$\begin{aligned}
\sigma_{rr} &= \frac{4\mu(\lambda+\mu)}{(\lambda+2\mu)}e_{rr} + \frac{2\mu\lambda}{(\lambda+2\mu)}e_{\varphi\varphi} - 2\mu\omega\Theta, \quad e_{zz} = \omega\Theta - \frac{\lambda(e_{rr}+e_{\varphi\varphi})}{(\lambda+2\mu)}, \\
\sigma_{rr} &= \frac{4\mu(\lambda+\mu)}{(\lambda+2\mu)}e_{\varphi\varphi} + \frac{2\mu\lambda}{(\lambda+2\mu)}e_{rr} - 2\mu\omega\Theta, \quad \Theta = \alpha(T - T_0), \quad \omega = \frac{(3\lambda+2\mu)}{(\lambda+2\mu)}
\end{aligned} \tag{5}$$

where λ , μ are Lame parameters, α is the coefficient of linear thermal expansion.

Within the framework of problem under consideration the radial and angular stresses must satisfy the equilibrium equation. Also corresponding values of the strains must satisfy to the continuity condition:

$$\sigma_{\varphi\varphi} = \left(r\sigma_{rr}\right)_{,r}, \quad d_{rr} = \left(rd_{\varphi\varphi}\right)_{,r}. \tag{6}$$

The Tresca's condition is selected as the yield criteria [6]:

$$\max\{|\sigma_{rr} - \sigma_{\varphi\varphi}|, |\sigma_{rr}|, |\sigma_{\varphi\varphi}|\} = 2k(T), \tag{7}$$

where $k(T)$ is the yield strength at the corresponding temperature. For further calculations, we assume the simple linear relation $k(T) = k_0(T_p - T)(T_p - T_0)$, where k_0 is the yield strength at the ambient temperature and T_p is the melting point.

Solution

Both the analytical solution [7] and the computational algorithms are existing for the heat equation (3) with boundary conditions (2). We assume that the temperature distribution is known.

The solution of the equilibrium equation (6) with boundary conditions (1) for the problem of thermoelasticity [6] has the form:

$$u_r(r,t) = \frac{\gamma r}{2} F_a(0,r,t) + c_1(t) r, \quad \sigma_{rr}(r,t) = \mu\big(2\omega c_1(t) - \gamma F_a(0,r,t)\big),$$

$$\sigma_{\varphi\varphi}(r,t) = \mu\big(2\omega c_1(t) + \gamma F_a(0,r,t) - \gamma\Theta(r,t)\big), \quad c_1(t) = -\frac{(\lambda+2\mu)F(0,R,t)}{2(\lambda+\mu)}, \tag{8}$$

$$\gamma = \frac{(3\lambda+2\mu)}{(\lambda+\mu)}, \quad F(r_0,r,t) = \frac{1}{r^2}\int_{r_0}^{r} \Theta(\rho,t)\rho d\rho.$$

Increasing of the temperature gradient leads to the satisfaction of the yield criteria (7) on the edge of the plate at the time t_p :

$$\sigma_{rr} - \sigma_{\varphi\varphi} = 2k. \tag{9}$$

At the time $t > t_p$ there is the plastic flow region $a_1(t) < r < R$, where $a_1(t)$ is the elastoplastic border. According to the associated flow rule the incompressibility condition follows from (9):

$$p_{rr} + p_{\varphi\varphi} = 0, \quad p_{zz} = 0. \tag{10}$$

The stresses in the plastic region could be found by integrating the system (6), (9):

$$\sigma_{rr}(r,t) = -G(a_1,r,t) + 2\mu\omega c_3(t), \quad \sigma_{\varphi\varphi}(r,t) = -G(a_1,r,t) + 2\mu\omega c_3(t) - 2k(r,t).$$

$$G(r_0,r,t) = \int_{r_0}^{r} \rho^{-1} k(\rho,t) d\rho. \tag{11}$$

Using the assumption (4), (10) and the Duhamel-Neumann Law (5) we obtain the differential equation for displacements in case of plastic strains existence:

$$u_{r,r} + \frac{u_r}{r} = \frac{\sigma_{rr} + \sigma_{\varphi\varphi}}{2\mu\omega} + 2\Theta. \tag{12}$$

Integrating the equation (12) we found:

$$u(r,t) = r\left(2F(a_1,r,t) - \frac{G(a_1,r,t)}{\mu\omega} + c_3(t) r + \frac{c_4(t)}{r^2}\right),$$

$$p_{rr}(r,t) = -p_{\varphi\varphi}(r,t) = \Theta(r,t) - 2F(a_1,r,t) - \frac{k(r,t)}{\mu\gamma}. \tag{13}$$

Note that the function of plastic deformation (13) doesn't depend on integration constants.

In the elastic deformation region $0 \le r < a_1(t)$ stress-strain state is determined by the relations (8) obtained previously with an accuracy to the new integration constants which together with the constants in the plastic region require its definition. For this it's needed to solve the system of linear equations in the form of boundary conditions (1) and continuity conditions of the radial stresses and displacements on the elastoplastic border $a_1(t)$ which is determined by the condition $p_{rr}(a_1,t) = 0$. Solution for integration constants has the form:

$$c_1(t) = \frac{\mu\gamma F(0,a_1,t) + 2G(a_1,R,t)}{2\mu\omega}, \quad c_3(t) = \frac{G(a_1,R,t)}{\mu\omega}, \quad c_4(t) = \frac{\gamma(1+\omega)a_1^2}{2\omega} F(0,a_1,t) \tag{14}$$

During the temperature field alignment, the non-reversible strains in the neighborhood of elastoplastic border continue increase whereas on the edge its rate becomes equal to zero:

$$p_{rr,t}(r,t_u)\big|_{r=R} = 0. \tag{15}$$

Relation (15) corresponds to the beginning of materials unloading, i.e. deformation process in which the yield condition (9) ceases to be satisfied. At the time $t > t_u$ the unloading region $a_2(t) \le r \le R$ exists. Displacements and stresses were obtained by solving of equation of equilibrium (6) taking into account irreversible strains [2]:

$$u_r(r,t) = \frac{\gamma r}{2} F(a_2,r,t) + \frac{(\lambda + 2\mu)r}{2(\lambda+\mu)} H(a_2,r) + c_5(t)r + \frac{c_6(t)}{r}, \quad H(r_0,r) = \int_{r_0}^{r} \frac{P(\rho)}{\rho} d\rho,$$
$$\sigma_{rr}(r,t) = \mu\gamma\left(-F_a(r,t) + H(a_2,r) + \frac{2\omega c_5(t)}{\gamma} - \frac{2c_6(t)}{\gamma r^2} \right), \tag{16}$$
$$\sigma_{\varphi\varphi}(r,t) = \mu\gamma\left(F_a(r,t) + H(a_2,r) + P(r) - \Theta(r,t) + \frac{2\omega c_5(t)}{\gamma} - \frac{2c_6(t)}{\gamma r^2} \right),$$

where $P(r)$ is the plastic strain captured at the given time on the unloading border which is defined by relation (15).

The stress–strain state in the plastic flow region $a_1(t) \le r < a_2(t)$ and the thermoelastic deformation region $0 \le r < a_1(t)$ is determined by the previously obtained relations with accuracy to the new integration constants. These constants with the constants in (16) were found from the system of linear equations describing the continuity of stresses and displacements on the regions boundaries.

The function $P(r)$ could be represented as the envelope of functions $p_{rr}(r,t)$ (13) with a parameter t. For the different values a_2 we found approximation $t(a_2)$ from the numerical solution of the equation $p_{rr,t}(a_2,t) = 0$. This approximation is an inverse function of the unloading boundary $a_2(t)$ and hence $P(r) = p_{rr}(r,t(a_2))$.

Results and discussion

The parameters corresponding to copper were used for calculations [8]:

$$\alpha = 17 \cdot 10^{-6} \frac{1}{{}^0C}, \quad R = 0.2m, \quad k_0 = 80 \cdot 10^6 Pa, \quad T_k = 520^0 C$$
$$\chi = 11.4 \cdot 10^{-5} \frac{m^2}{s}, \lambda = 91.2 \cdot 10^9 Pa, \mu = 42.9 \cdot 10^9 Pa, T_0 = 20^0 C \tag{17}$$

The solution was found for different values of parameter x (heating rate) in (2). The plastic flow does not appear at small values of x and small temperature gradient. The appearance of the plastic flow in the neighborhood of the plate edge was observed with increasing of parameter x. Temperature stresses decrease and unloading region appears in the temperature gradient alignment. The unloading border eventually overtakes the elastoplastic border. In case of high values of x after the complete unloading of material, the condition (9) with the opposite sign in front of the yield point is satisfied on the edge of the plate. This fact means the development of plastic flow when the plastic deformation increases in the opposite direction. Thereby value decreasing of the residual strain was accumulated during heating process. The occurrence of repeated plastic flow is caused by high residual stresses generated as a result of temperature equalization and also significant decrease of the yield point.

The thermal stresses at the moment of the material unloading are presented on Fig. 1.

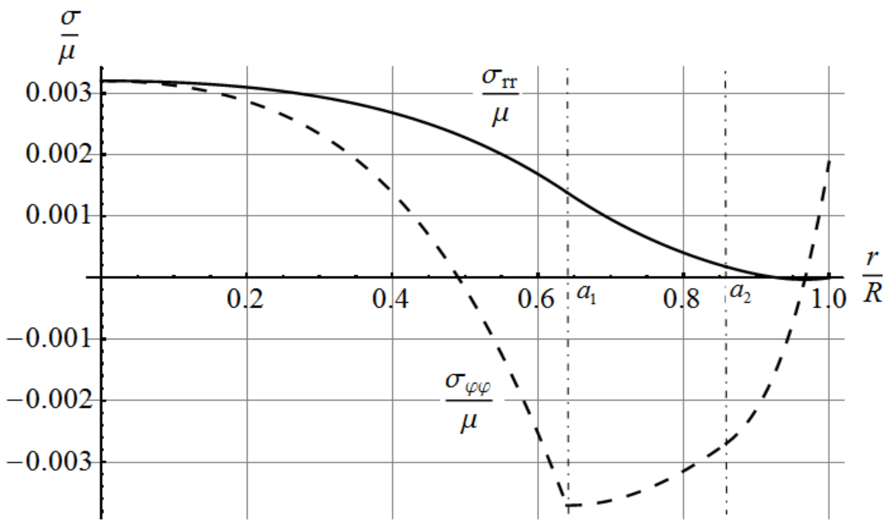

Figure 1. The residual stresses distribution. The plastic flow border a_1 and the unloading border a_2 are marked by vertical dashed lines.

Note that the level of stresses is independent of the current temperature and determined by the temperature gradient level. Consequently, the distribution of stresses at complete heating of the plate to a maximum temperature coincides with the stress field at complete cooling.

Conclusion

The problem of unsteady thermal action on the thin circular plate has been considered. This physical process has been mathematically proposed as a quasi-static process of the uniform thermal expansion of the plate. Generalized Prandtl-Reuss thermoelastoplastic model was used. The effect of non-stationary temperature gradient on the residual stresses field formation was investigated in condition of dependence of the yield stress on temperature. The resulting system was analytically integrated. The border of irreversible deformation domain and unloading domain were computed. The level of the residual stresses was calculated.

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