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On Finite Displacement of an Elastoviscoplastic Material in a Gap between Two Rigid Coaxial Cylindrical Surfaces

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Abstract—In the framework of the theory of large deformations, we obtain the solution of a boundary value problem on the flow of an elastoviscoplastic material in a gap between two rigid coaxial cylindrical surfaces under pressure drop changing with time. It is assumed that slip of the material is possible on both surfaces. We consider reversible deformation, the development of viscoplastic flow under the increasing and constant pressure drop, deceleration of the flow under the decreasing pressure drop, and the unloading of the medium.

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The exact solutions to the problems of antiplane plastic flow within the framework of the Shvedov–Bingham model have been repeatedly obtained [1–4]. Also, some fairly universal methods have been developed for calculation of viscoplastic flows [5, 6]. The abandonment of the assumption on the non-deformability of the medium in stagnant zones or the medium forming rigid nuclei leads to a significant complication of the mathematical simulation of its flows. Deformations in such domains are reversible; therefore, the formulation of the boundary-value problems must be carried out in terms of displacements. But, in the flow domains, the problem is solved in terms of displacement velocities. The fulfilment of the continuity conditions for velocities and stresses on the boundaries of the domains is insufficient and can lead to erroneous solutions [7]. Therefore, it is necessary that the continuity conditions for displacements be fulfilled, although to calculate the displacements in the flow domains is sometimes difficult [8].

Simulation of the flow process should be carried out in large deformations since at least irreversible deformations cannot be considered small. Rather many models have been proposed of large elastoplastic deformations beginning from the first geometrically consistent model [9]. Let us note some domestic works [10–13]. We will use the mathematical model of [14] that is described in detail in [15]. This model meets the classical requirements to the elastoplastic model: during the unloading process, the irreversible deformations change in the same way as in the case of rigid body motion, the stresses in the medium are completely determined by reversible deformations, the unloaded state does not depend on the path of unloading in the stress space. These requirements are not mandatory, but their formulation as assumptions greatly simplifies the model of large elastic-plastic deformations and allows us to obtain solutions of boundary value problems related to elastoplastic and elastoviscoplastic [16–19] deformation of materials acquiring large deformations, even including some exact solutions.

Below we construct the solution of the problem on the flow of an elastoviscoplastic medium in a cylindrical layer under conditions of a varying pressure drop. In [18, 19] the problems were considered, close in formulation, on the plastic flow of a medium in a cylindrical tube under the influence of varying pressure drop and in a cylindrical layer when the inner boundary surface moves.

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1. BASIC MODEL RELATIONS

In the rectangular Cartesian system of the Euler spatial coordinates x_i, the kinematics of the medium [14] is determined by the dependences

$$
\begin{gathered}
d_{ij} = e_{ij} + p_{ij} - \frac{1}{2}e_{ik}e_{kj} - e_{ik}p_{kj} - p_{ik}e_{kj} + e_{ik}p_{ks}e_{sj},\\
\frac{De_{ij}}{Dt} = \varepsilon_{ij} - \varepsilon^p_{ij} - \frac{1}{2}\big((\varepsilon_{ik} - \varepsilon^p_{ik} + z_{ik})e_{kj} + e_{ik}(\varepsilon_{kj} - \varepsilon^p_{kj} + z_{kj})\big),\\
\frac{Dp_{ij}}{Dt} = \varepsilon^p_{ij} - p_{ik}\varepsilon^p_{kj} - \varepsilon^p_{ik}p_{kj}, \qquad \frac{Dn_{ij}}{Dt} = \frac{dn_{ij}}{dt} - r_{ik}n_{kj} + n_{ik}r_{kj},\\
\varepsilon_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}), \qquad v_i = \frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_{i,j}v_j, \qquad u_{i,j} = \frac{\partial u_i}{\partial x_j},\\
r_{ij} = w_{ij} + z_{ij}(\varepsilon_{sk}, e_{sk}), \qquad w_{ij} = \frac{1}{2}(v_{i,j} - v_{j,i}).
\end{gathered}
\tag{1}
$$

Here, d_{ij} are the components of the Almansi strain tensor; $e_{ij} - 0.5e_{ik}e_{kj}$ and p_{ij} are its invertible and noninvertible components; u_i and v_i are the components of displacement vectors and velocities of the points of the medium; $\dfrac{D}{Dt}$ is the used objective tensor derivative with respect to time, which is given for an arbitrary tensor n_{ij}; and ε^p_{ij} (source in the transport equation for the irreversible deformation tensor) are the components of the tensor of the plastic strain rates. The presence of a nonlinear component of the rotation tensor z_{ij}, which is given in [15], is connected with the fulfillment of the invariance requirement for the plastic deformation tensor p_{ij} in the unloading processes.

We consider the material incompressible and take into account the condition of the density of the free energy distribution independent of irreversible deformations. Then we obtain the analog of Murnaghan's formula [14]:

$$
\begin{gathered}
\sigma_{ij} = -P\delta_{ij} + \frac{\partial W}{\partial d_{ik}}(\delta_{kj} - 2d_{kj}) \quad \text{for} \quad p_{ij} \equiv 0, \quad i,j,k = 1,2,3,\\
\sigma_{ij} = -P_1\delta_{ij} + \frac{\partial W}{\partial e_{ik}}(\delta_{kj} - e_{kj}) \quad \text{for} \quad p_{ij} \neq 0,\\
W = -2\mu J_1 - \mu J_2 + bJ_1^2 + (b-\mu)J_1J_2 - \chi J_1^3 + \dots,\\
J_k = \begin{cases} L_k, & \text{for} \quad p_{ij} \equiv 0,\\ I_k, & \text{for} \quad p_{ij} \neq 0, \end{cases} \qquad L_1 = d_{kk}, \qquad L_2 = d_{ik}d_{kj},\\
I_1 = e_{kk} - 0.5e_{st}e_{ts}, \qquad I_2 = e_{st}e_{ts} - e_{sk}e_{kt}e_{ts} + 0.25e_{sk}e_{kt}e_{tn}e_{ns},
\end{gathered}
\tag{2}
$$

where σ_{ij} are the components of the Euler–Cauchy stress tensor, P and P_1 are the additional hydrostatic pressures, $W = W(e_{ij})$ is the elastic potential, μ, b, and χ are the constants of the material, and δ_{ij} is the Kronecker symbol.

As the plastic potential, we use Tresca's condition

$$\max|\sigma_i - \sigma_j| = 2k + 2\eta\max\left|\varepsilon^p_n\right| \tag{3}$$

in which σ_i and ε^p_n are the principal values of the tensors of stresses and rates of plastic deformations, k is the yield stress, and η is the coefficient of viscosity.

The rates of irreversible deformations are related to stresses by the associated law of plastic flow:

$$\varepsilon^p_{ij} = \lambda\frac{\partial f}{\partial \sigma_{ij}}, \qquad f\left(\sigma_{ij}, \varepsilon^p_{ij}\right) = k, \qquad \lambda > 0. \tag{4}$$