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Residual Stresses in AM Fabricated Ball During a Heating Process

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Abstract. The present study is devoted to the problem of residual stresses calculation in AM fabricated ball during heating. Strains of the ball are assumed to be small, which allows to use the apparatus of the theory of thermoelastoplastic akin to Prandtl and Reuss. The problem of the evolution of the field of residual stresses in the ball at a given temperature on its external border is solved. The heat conduction equation and the equilibrium equations may be independently integrated when the hypothesis of the insignificance of the coupled effects of thermal and mechanical processes is adopted. The fields of residual stresses and displacements are computed.

INTRODUCTION

The studies of the stress-strain state of multilayer (composite) structures are actual problems of modern Continuum Mechanics and Mechanics of the Technological Processes [1, 2, 3, 4, 5, 6, 7, 8]. The influence of temperature effects in such processes naturally allows one to accurate prescribe the formation of residual stresses and deformations significantly imposing the restrictions on the exploitation and durability of structures. One such example is the problem of mathematical modelling of the technological operation of hot fitting [9, 10, 11, 12].

The initial thermal expansion plays a great role in the formation of residual stresses and deformations and, consequently, influences on the strength characteristics of the manufactured product (see for example [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]).

The process of adding new parts of the material can be considered as a process of discrete material growth used in the technology of additive manufacturing. The mechanics of growing bodies [25, 26, 27] can be considered as a theoretical basis for solving such problems. In [28, 29, 30], boundary value problems of the growth of heavy viscoelastic bodies were solved with the gravitational forces presence. The thermal state of a growing viscoelastic sphere was discussed in [31].

TWO-LAYERED SPHERE DEFROMATION

Throughout the paper we will use the conventional Prandtl–Reuss elastic plastic model [1] generalized on thermal effects. Consider a spherical layer under the number $x = 1$ with an internal R_0 and external R_1 radii and being in a free state at referential temperature $T = T_1$. Suppose that at some time $t = t_1$ the uniformly heated to the temperature $T = T_2$ second spherical layer with radii R_1 and R_2 is attached to the outer surface of the first layer. We assume the

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conditions of an ideal thermal contact on the layers contact surface, and assume that the cooling of the bilayer material occurs as a result of heat flux through the free surfaces. Then it can be assumed that the temperature of the outer layer decreases uniformly to the temperature of the continuum T_1. At the same time, the temperature change of the inner layer does not have a significant effect on the stress-strain state formation. The thermal stresses are increased in both layers as a result of cooling caused by restrictions of free thermal compression on the inner contact surface. Let's introduce the following notation: m is the number of layers, x is the actual layer number ($x = 1..m$), then R_{x-1}, R_x is the size of each layer before its adding, T_x is the initial temperature of the layer before its adding.

We consider the governing equations of the problem in a one-dimensional statement in a spherical coordinate system. In the frameworks of the infinitesimal deformations theory the relations between the components of the deformation tensor d_{ij} and the radial component of the displacement vector u_r are furnished by

$$d_{rr} = e_{rr} + p_{rr} = \partial u_r/\partial r, \quad d_{\varphi\varphi} = d_{\vartheta\vartheta} = e_{\varphi\varphi} + p_{\varphi\varphi} = u_r/r. \tag{1}$$

The relations between the stress tensor components σ_{ij} and strain ones e_{ij} for the isotropic material are subject to the Duhamel–Neumann law

$$\begin{gathered} \sigma_{rr} = (\lambda + 2\mu)e_{rr} + 2\lambda e_{\varphi\varphi} + q\Delta_i, \\ \sigma_{\varphi\varphi} = \sigma_{\vartheta\vartheta} = 2(\lambda + \mu)e_{\varphi\varphi} + \lambda e_{rr} + q\Delta_i. \end{gathered} \tag{2}$$

wherein λ, μ are the Lame parameters, $q = (3\lambda + 2\mu)$, $\Delta = \alpha(T - T_1)$ is the thermal expansion coefficient, α is the linear thermal expansion coefficient.

The components of the stress tensor satisfy the equation of equilibrium

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{2(\sigma_{rr} - \sigma_{\varphi\varphi})}{r} = 0. \tag{3}$$

The system of equations (1–3) are closed and allows us to derived solution for stresses and displacements in an arbitrary layer for the general case in the absence of irreversible deformations ($p_{ij} = 0$)

$$\sigma_{rr}(x,r) = A_{mx} + \frac{B_{mx}}{r^3}, \quad \sigma_{\varphi\varphi}(x,r) = \sigma_{\vartheta\vartheta}(x,r) = A_{mx} - \frac{B_{mx}}{2r^3}, \quad u_r(x,r) = -r\Delta_x + \frac{rA_{mx}}{(3\lambda + 2\mu)} - \frac{B_{mx}}{4\mu r^2}. \tag{4}$$

A_{mx}, B_{mx} are the integration constants that depend on the boundary conditions and the layers number. The boundary conditions of the considered problem are read by the following equations

$$\begin{gathered} \sigma_{rr}(1,R_0) = 0, \quad \sigma_{rr}(m,R_m) = 0, \\ \sigma_{rr}(x,R_x) = \sigma_{rr}(x+1,R_x), \quad u_r(x,R_x) = u_r(x+1,R_x). \end{gathered} \tag{5}$$

Note that the system of boundary conditions (5) is valid only for calculation of the reversible deformation process. Since it does not take into account the deformation prehistory. Thus, in the case of thermoelastic equilibrium of a two-layered sphere ($m = 2$) for the integration constants we can obtain

$$\begin{aligned} A_{21} &= \frac{4\mu(3\lambda + 2\mu)(R_2^3 - R_1^3)\Delta_2}{3(\lambda + 2\mu)(R_2^3 - R_0^3)}, \quad B_{21} = -\frac{4\mu(3\lambda + 2\mu)R_0^3(R_2^3 - R_1^3)\Delta_2}{3(\lambda + 2\mu)(R_2^3 - R_0^3)}, \\ A_{22} &= \frac{4\mu(3\lambda + 2\mu)(R_1^3 - R_0^3)\Delta_2}{3(\lambda + 2\mu)(R_2^3 - R_0^3)}, \quad B_{22} = -\frac{4\mu(3\lambda + 2\mu)R_2^3(R_1^3 - R_0^3)\Delta_2}{3(\lambda + 2\mu)(R_2^3 - R_0^3)}, \end{aligned} \tag{6}$$

Hereafter the resulting equations for the integration constants are not given in view of their simple derivations from the system of linear equations (boundary conditions) and the cumbersome resulting equations.

Calculations of thermoelastic equilibrium parameters show that the absolute maximum values of stresses occur on the inner surfaces of both layers. The stress level is proportional to the difference $\Delta_x - \Delta_{x+1}$. Consequently, for a certain value of Δ_2, the absolute value of the shear stress $|\sigma_{rr} - \sigma_{\varphi\varphi}|$ determining the permissible regimes of the reversible deformation processes can reach values at which the plastic flow of the material is possible.

Stresses inside the plastic flow domain satisfy the yield criterion

$$\sigma_{rr}(x,r) - \sigma_{\varphi\varphi}(x,r) = 2s_x k, \tag{7}$$

wherein $s_x = \mathrm{sgn}(\sigma_{rr}(x,R_{x-1}) - \sigma_{\varphi\varphi}(x,R_{x-1}))$, k is the yield stress.

The equation of equilibrium integration (3) under the boundary condition (7) allows us to obtain equations for stresses, displacements and irreversible deformations inside the plastic flow domain ($R_{x-1} < r < v_x$)

$$\begin{gathered}\sigma_{rr}(x,r) = F_{2x} - 4s_x \ln(r),\\ \sigma_{\varphi\varphi}(x,r) = \sigma_{\vartheta\vartheta}(x,r) = F_{2x} - 4s_x \ln(r) - 2s_x k,\\ u_r(x,r) = \frac{G_{2x}}{r^2} - \frac{4krs_x \ln(r)}{(3\lambda+2\mu)} + r\left(\frac{F_{2x}}{(3\lambda+2\mu)} - \Delta_x\right),\\ p_{rr}(x,r) = -2p_{\varphi\varphi} = -2p_{\vartheta\vartheta} = -\left(\frac{2G_{2x}}{r^3} + \frac{2s_x k(\lambda+2\mu)}{\mu(3\lambda+2\mu)}\right).\end{gathered} \tag{8}$$

The presence of a plastic flow leads to a change in the integration constants equations (6) and the appearance of new constants G, F. To find a new set of constants, the conditions (5) must be supplemented by the conditions for the equality of radial stresses and displacements at the boundaries v_x separating the domain of plastic flow from the one of reversible deformation. After this, the positions of the elastoplastic boundaries can be found as the numerical solution of the system of equations $p_{rr}(x, v_x) = 0$. The heat transfer leads to the formation of the final residual stresses distribution. In this case, the elastoplastic boundaries hold their positions inside the domains with accumulated plastic deformations and the yield criterion (7) is valid. Thus, the stress state of the material corresponds to the regime of neutral loading.

Fig. 1 shows the distribution of residual stresses in the two-layers material in the case of plastic flow in both layers.

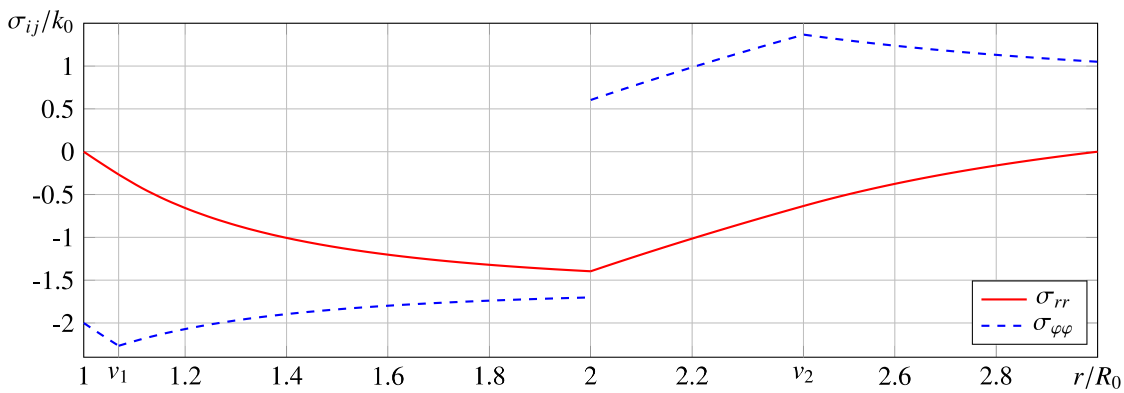

FIGURE 1. Residual stresses of a two layered sphere

THREE-LAYERED SPHERE DEFROMATION

In this section consider the problem of adding a third layer $x = 3$ with internal and external radii $R_2 < r < R_3$ heated to the initial temperature $T_3 = T_2$ to the formed two-layered sphere considered in previous section. Because of the smallness of the deformations, The displacement of the outer surface of the cooled second layer can be neglected by changing the displacements and assume the size of the composite sphere by $R_0 < r < R_2$. In order to determine how the heat transfer and the thermal compression of the outer third layer processes affect to the change in residual stresses in the resulting composite sphere we will assume that the material is everywhere deformed elastically with the accumulated irreversible deformations. Calculation of the thermoelastic state of the material will allow us to determine the stress level causing the plastic flow process.

Denote the function of the irreversible deformation by

$$P(x,r) = \begin{cases} -\left(\dfrac{2G_{2x}}{r^3} + \dfrac{2s_x k(\lambda+2\mu)}{\mu(3\lambda+2\mu)}\right), & R_{x-1} < r < v_x; \\ 0, & v_x < r < R_x \end{cases} \tag{9}$$

Then we find equations for the residual stresses and displacements in virtue of irreversible deformation (9)

$$\begin{aligned} \sigma_{rr}(x,r) &= \frac{2\mu(3\lambda+2\mu)}{(\lambda+2\mu)} I(x,r) + M_{3x} + \frac{N_{3x}}{r^3}, \\ \sigma_{\varphi\varphi}(x,r) &= \frac{\mu(3\lambda+2\mu)}{(\lambda+2\mu)}(2I(x,r)+P(x,r)) + M_{3x} + \frac{N_{3x}}{r^3}, \\ u_r &= \frac{2\mu r}{(\lambda+2\mu)} I(x,r) - r\Delta_i + \frac{rM_{3x}}{(3\lambda+2\mu)} - \frac{N_{3x}}{4\mu r^2}, \\ I(x,r) &= \int_{R_{x-1}}^{r} \frac{P(x,\rho)}{\rho} d\rho. \end{aligned} \tag{10}$$

Note the equations (10) correspond to the distributions shown on Fig. 1.

The stress state of the third layer is given by the equations (4) with integration constants A_{33}, B_{33}. It was found that with thermal compression and cooling of the third layer the intensity of shearing stresses decreases at each point of the second layer. Thus, the material stress strain state in the domain $R_1 < r < v_2$ passes from the state of neutral loading to the unloading state. At the same time, the conditions of the continuity of the circumferential stresses are satisfied on the inner boundary of the third layer. The material stress strain state in the first layer $R_0 < r < v_1$ changes from the state of neutral loading to the plastic flow with a change of the position of the elastoplastic boundary. Fig. 2 illustrates the stress distributions in a three-layer thermoelastoplastic sphere.

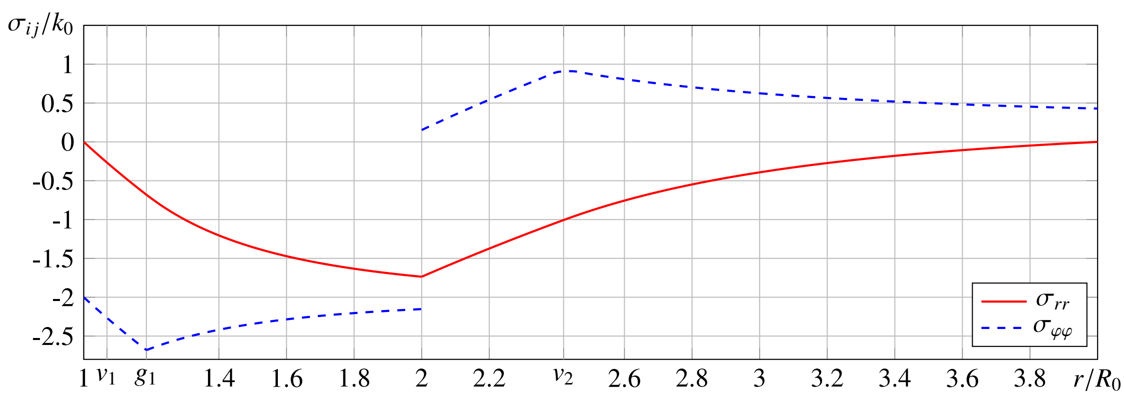

FIGURE 2. Residual stresses of a three-layered sphere. g_1 is a new location of the elastic plastic border in the inner sphere layer

CONCLUDING RESULTS DISCUSSION

On the basis of this solution, it can be concluded that each subsequent adding layer having the same heating temperature leads to the propagation of plastic flow in the inner layer. Moreover, the continuity conditions of the circumferential stresses are satisfied on each contact surface $R_x (x > 0)$. The jumps in the circumferential stresses during the addition of the third layer are possible in two cases:

1. for a different levels of initial heating temperatures of the second and third adding layers and consequently a different thermal expansion rates during cooling;

2. if the size of the inner surface of the third heated layer is equal to the size of the outer surface of the second layer during cooling.

Both cases are equivalent to each other with the thermal deformation gradient $\Delta_2 - \Delta_3$ arising on the contact surface. The accumulation of plastic deformation on the inner surface of the first layer leads to a rearrangement of the deformation causing the decreasing of the second layer level stress and the material unloading.

When the subsequent layers are attached, further deformations are accumulated on the inner surface of the composite ball, while in the interlayer zone the conditions of compatibility of deformations (continuity of stresses) are preserved. Therefore, part of the stackable multilayer material can be considered as solid with dimensions $R_1 < r < R_m$.

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