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Residual Stresses in Blood Vessel Wall During Atherosclerosis

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Abstract. The present study is devoted to the computational problem of the residual stresses inside human blood vessel wall during atherosclerosis. The blood vessel (artery) wall is simulated by the thick-walled very long circular cylinder. The governing and constitutive equations of mechanics of surface growth solids are reminded for thick-walled solid case. The boundary value problem of the surface growth of an elastic thick-walled vessels is stated, solved and analyzed. The analytical solution is obtained in terms of velocities of stress strain state parameters. The condition of thickness allows us to study finite displacements of cylinder surfaces by means of infinitesimal deformations. The residual stresses caused by the influence of internal pressure are computed according to various initial growth tension in the adding layer.

PRELIMINARY REMARKS

Atherosclerosis is a multifactorial disease manifested in the accumulation in the artery wall of protein-lipid components, collagen, inflammatory cells. The last stages is characterized a sharp decrease in blood flow, due to a decrease in the lumen of the artery. Atherosclerosis is the leading cause of cardiovascular diseases (myocardial infarction, strokes, etc.) and as a consequence the leading worldwide cause of death [1]. Atherosclerosis is a chronic disease that has been developing asymptomatically for decades. The first visible clinical signs can be seen only in the last stages of atherosclerosis, when the greater part of the lumen of the vessel is occluded [2, 3].

The process of formation of atherosclerotic plaques occurs in following stages:

- The appearance of lipid spots at the first stage is due to the deposition of lipid-protein complexes of blood plasma in a thin layer of the inner shell of the arteries [4]. Later on these spots can develop atherosclerotic plaques. The accumulation of lipoproteins in the inner shell of the artery is promoted by an increased concentration of cholesterol in the plasma, damaged endothelium, etc [5].
- Lipoprotein complexes are partially bound to the intercellular substance. Then there is oxidation, which causes local inflammation. Inflammation causes the attachment of blood plasma leukocytes. Phagocytizing lipoproteins, macrophages are converted into xantom cells. This contributes to the thickening of the intima, the accumulation in it the components of blood plasma, collagen and inflammatory cells. A lipid-rich atheromatous mass appears after xantom cells die [6].
- Initially, the plaque slowly grows almost without narrowing the lumen of the vessel. However, over time, its growth accelerates and it significantly narrows the lumen of the vessel.
- In the late stages of atherosclerosis, small ruptures appear on the surface of the plaques causing adhesion of the blood and fibrin elements, which narrow the lumen even more. It is the main mechanism of thrombi formations [7]. Atherosclerosis affects vessels of different calibers, but mainly arteries of large and medium caliber $1-3cm$ (they constitute $90-95\%$ of the lesion).

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The main research methods used in medicine, allowing to assess the progress of the degree of artery lumen narrowing due to atherosclerosis:

1. duplex ultrasound of blood vessels allows to detect the volume of blood and artery damage [8];
2. magnetic resonance angiography allows to estimate the sizes of atherosclerotic masses and the degree of narrowing of the vessel lumen [9];
3. computed tomography allows to obtain layered ”slices” of the artery, to estimate the degree of occlusion with atherosclerotic masses. It is often used together with angiography;
4. proper angiography makes it possible to determine the volume of the blood stream in the vessel after the administration of the radiopaque substance [10].

The pathological growth of blood vessel wall can be described in some cases by surface growth mechanics technique. At present study we will focused on the processes of surface growth of thin-walled vessels. We use the ideas of the mechanics of growing solids developed in [11, 12, 13, 14]. Some problems in frameworks of the thermoelastoplasticity are studied for the same symmetries conditions in [15, 16, 17, 18, 19, 20, 21, 22, 23]. The principal variables of the boundary value problem for a growing body are the stress rate tensor, the strain rate tensor and the velocity vector. On the surface of growth we set a specific boundary condition depending on the curvature tensor of the growth surface and the tension and inflow rates of the incremented elements.

Some problems for an elastic thick-walled surface-growing cylinder are considered at present work. The condition of thickness allows us to study finite displacements of cylinder points under the condition of small deformations. This, in particular, makes it possible to solve the problem with exact boundary conditions on a moving surface.

GOVERNING EQUATIONS OF SURFACE GROWTH THEORY

Throughout the paper we will use the conventional linear elastic model [11] generalized on growing body mechanics effects. In this case the velocity of deformation can be neglected as compared with the known the growth surface velocity. The boundary value problem can be formulated in terms of velocities in the following form the equilibrium equation

$$\nabla \cdot \mathbf{S} = \mathbf{0}, \tag{1}$$

the constitutive equations

$$D = \frac{1}{2}[\boldsymbol{\nabla}\mathbf{v} + (\boldsymbol{\nabla}\mathbf{v})^T], \quad \mathbf{S} = 2\mu\mathbf{D} + \lambda(\mathbf{D})\mathbf{1}, \tag{2}$$

the boundary conditions

$$\begin{gathered} \mathbf{x} \in S_1: \quad \mathbf{n} \cdot \mathbf{S} = \frac{\partial \mathbf{p}_0}{\partial t}, \qquad \mathbf{x} \in S_2: \quad \mathbf{v} = \frac{\partial \mathbf{u}_0}{\partial t}, \\ \mathbf{x} \in S^*(t): \quad \mathbf{n} \cdot \mathbf{S} = -\frac{s_n}{G}(\boldsymbol{\tau}_s : \mathbf{L})\, \mathbf{n}, \quad s_n = \mathbf{n} \cdot \mathbf{v}, \\ \mathbf{v} = \mathbf{v}_{\text{gr}}, \quad t = \tau^*(\mathbf{x}), \end{gathered} \tag{3}$$

the recovery formulae

$$\begin{aligned} \boldsymbol{\sigma}(\mathbf{x}, t) &= \boldsymbol{\sigma}(\mathbf{x}, \tau^*(\mathbf{x})) + \int_{\tau^*(\mathbf{x})}^{t} \mathbf{S}(\mathbf{x}, \tau)\, ds, \\ \mathbf{u}(\mathbf{x}, t) &= \mathbf{u}(\mathbf{x}, \tau^*(\mathbf{x})) + \int_{\tau^*(\mathbf{x})}^{t} \mathbf{v}(\mathbf{x}, \tau)\, ds. \end{aligned} \tag{4}$$

where $\mathbf{S}$ is the stress rate tensor; $\boldsymbol{\nabla}$ is the three-dimensional Hamiltonian differential operator (the nabla symbol); $\mathbf{p}_0$, $\mathbf{u}_0$ are given vectors of surface forces and strains; $\mathbf{n}$ is the unit normal vector to the solid surface; $\mathbf{v}$ is the velocity vector; $\boldsymbol{\tau}_s$ is the 2D tensor of the deposited elastic surface tension; $\mathbf{L}$ is the 2D tensor of growth surface curvature; $\mathbf{D}$ is the stretch tensor; $\mathbf{v}_{\text{def}}$ is the unknown velocity of the boundary due to deformation of a solid; $\mathbf{v}_{\text{gr}}$ is the prescribed velocity of growth.

This boundary value problem (1)–(4) is mathematically identical to the boundary value problem of the theory of elasticity for small deformations (Lame problem) and most adequate results have been obtained in the framework of this version of the theory (see for example [24, 25, 26, 27, 28]).

BOUNDARY VALUE PROBLEMS STATEMENT AND SOLUTION

Consider a very long hollow elastic circular cylinder with internal and external radii R_1 and R_2 respectively. The lateral surface of the cylinder is subjected to a stationary loading pressure

$$\sigma_{rr}(R_1) = p_1, \quad \sigma_{rr}(R_2) = -p_2. \tag{5}$$

Suppose at time $t = 0$ on the inner surface of the cylinder the new material is adding with the velocity $v = v(t)$ and the circumferential tension of the adding layer $\tau(t)$ when the boundary conditions (3) for the present case can be furnished by

$$S_{rr}(R_1(t)) = -\frac{v(t)\tau(t)}{R_1(t)}, \quad R_1(t) = R_1 - v(t)t, \quad S_{rr}(R_2) = 0. \tag{6}$$

The solution of the considered problem can be obtained in following form for stresses

$$\begin{aligned} \sigma_{rr} &= A + \frac{B}{r^2} + \int_0^t \left(X(s) + \frac{Y(s)}{r^2}\right) ds, \\ \sigma_{\varphi\varphi} &= A - \frac{B}{r^2} + \int_0^t \left(X(s) - \frac{Y(s)}{r^2}\right) ds, \ \sigma_{zz} = \frac{\lambda_a}{(\lambda_a + \mu_a)}\left(A + \int_0^t X(s) ds\right). \end{aligned} \tag{7}$$

and translational displacements after integrating on time

$$u_r = \frac{Ar}{2(\lambda_a + \mu_a)} - \frac{B}{2\mu_a r} + \left(\int_0^t \frac{X(s)r}{2(\lambda_p + \mu_p)} - \frac{Y(s)}{2\mu_p r}\right) ds. \tag{8}$$

The relations for an undefined functions A and B are derived by the solution of the boundary conditions system (5) in forms

$$A = -\frac{p_1 R_1^2 + p_2 R_2^2}{R_2^2 - R_1^2}, \quad B = \frac{R_1^2 R_2^2 (p_1 + p_2)}{R_2^2 - R_1^2}. \tag{9}$$

Unknown functions $X(t)$ and $Y(t)$ can be computed by the solution of the system (6) in following form

$$X(t) = \frac{R_1(t)v(t)\tau(t)}{R_2^2 - R_1^2(t)}, \quad Y(t) = -\frac{R_1(t)v(t)\tau(t)R_2^2}{R_2^2 - R_1^2(t)}. \tag{10}$$

CONCLUDING REMARKS

Numerical calculations were made for the following dimensionless parameters: values measured in pascals (σ_{ij}, λ_a, μ_a, τ, p_1, p_2) are dimensioned by the the Young's modulus of the material of the artery; ratio of Young's moduli atherosclerotic plaque material and artery is assumed equal to (2/3); therefore, for Lame parameters of the atherosclerotic plaque material we have formulae $\lambda_p = (2/3)\lambda_a$, $\mu_p = (2/3)\mu_a$; values measured in meters (r, u_r, R_1, R_2) are dimensioned by the value of the outer radius of the artery R_2. The numerical experiments were carried out for various values of the growing layer size $R_1(t)$ and the growth layer tension level $\tau(t)$. The growth tension level τ has a important influence on the formation of the final stress-strain state of the material for the case of constant velocity $v(t) = const$ and tension $\tau(t) = const$. The stresses can be vanished on the inner surface $R_1(t)$ for a specific value of growth layer tension τ.

The residual stresses caused by the influence of internal pressure can be computed in the absence of initial growth tension in the adding layer by formulae (7). The residual stresses radial distribution caused by the high level of the initial growth tension during the narrowed vessel lumen are calculated. Thus, it is possible to conclude, that on the one hand, with correct initial parameters of the growing process, it is possible to optimise a minimum effect of the blood pressure on vessel wall. To another hand, a certain growth regime leads to significant irreversible deformation and crack of vessel wall.

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REFERENCES

[1] Y. T. Lee, H. Y. Lin, Y. W. F. Chan, K. H. C. Li, O. T. L. To, B. P. Yan, T. Liu, G. Li, W. T. Wong, W. Keung, *et al.*, Lipids in health and disease **16**, p. 12 (2017).

[2] M. Tennant and J. K. McGeachie, ANZ Journal of Surgery **60**, 747–753 (1990).

[3] J. Folkman, Annu. Rev. Med. **57**, 1–18 (2006).

[4] S. Lim and S. Park, BMB reports **47**, p. 1 (2014).

[5] I. Tabas, G. García-Cardeña, and G. K. Owens, J Cell Biol **209**, 13–22 (2015).

[6] D. Wang, Z. Wang, L. Zhang, and Y. Wang, Mediators of inflammation **2017** (2017).

[7] B. Furie and B. C. Furie, New England Journal of Medicine **359**, 938–949 (2008).

[8] L. Santoro, P. M. Ferraro, A. Flex, A. Nesci, G. De Matteis, A. Di Giorgio, V. Zaccone, G. Gambaro, A. Gasbarrini, and A. Santoliquido, Hypertension Research **39**, p. 868 (2016).

[9] G. Wu, J. Yang, T. Zhang, J. N. Morelli, S. Giri, X. Li, and W. Tang, Journal of Cardiovascular Magnetic Resonance **18**, p. 71 (2017).

[10] B. L. Kietselaer, C. P. Reutelingsperger, G. A. Heidendal, M. J. Daemen, W. H. Mess, L. Hofstra, and J. Narula, New England Journal of Medicine **350**, 1472–1473 (2004).

[11] A. V. Manzhirov, "Mechanics of growing solids and phase transitions," in *Key Engineering Materials*, Vol. 535 (Trans Tech Publ, 2013), pp. 89–93.

[12] A. V. Manzhirov, Procedia Engineering **173**, 11–16 (2017).

[13] A. V. Manzhirov, "Mechanical design of am fabricated prismatic rods under torsion," in *MATEC Web of Conferences*, Vol. 95 (EDP Sciences, 2017) p. 12002.

[14] A. V. Manzhirov, "Application of mechanics of surface growth to problems of additive manufacturing," in *Mathematics and Computers in Sciences and in Industry (MCSI), 2017 Fourth International Conference on* (IEEE, 2017), pp. 86–91.

[15] A. A. Burenin, E. P. Dats, S. Mokrin, and E. V. Murashkin, Bulletin of Yakovlev Chuvash State Pedagogical University. Series: Mechanics of Limit State **16**, 22–28 (2013).

[16] A. A. Burenin, E. P. Dats, and E. V. Murashkin, mechanics of Solids **49**, 218–224 (2014).

[17] E. Dats, E. Murashkin, and N. Stadnik, Procedia Manufacturing **7**, 427–434 (2017).

[18] E. Dats, E. Murashkin, and N. Stadnik, Procedia Engineering **173**, 891–896 (2017).

[19] E. P. Dats, E. V. Murashkin, and N. K. Gupta, Procedia IUTAM **23**, 187–200 (2017).

[20] E. Dats, E. Murashkin, and N. Stadnik, IAENG International Journal of Applied Mathematics **47**, 261–264 (2017).

[21] E. Murashkin and E. Dats, Mechanics of Solids **52**, 495–500 (2017).

[22] E. Murashkin, E. Dats, and V. Klindukhov, "Numerical analysis of the elastic-plastic boundaries in the thermal stresses theory frameworks," in *Journal of Physics: Conference Series*, Vol. 937 (IOP Publishing, 2017) p. 012030.

[23] E. Murashkin and E. Dats, "Coupled thermal stresses analysis in the composite elastic-plastic cylinder," in *Journal of Physics: Conference Series*, Vol. 991 (IOP Publishing, 2018) p. 012060.

[24] A. Manzhirov, Journal of Applied Mathematics and Mechanics **49**, 777–782 (1985).

[25] A. Manzhirov, Lecture Notes in Engineering and Computer Science **2218(1.1)**, 710–714 (2015).

[26] A. V. Manzhirov, M. N. Mikhin, and E. V. Murashkin, Vestnik Samarskogo Gosudarstvennogo Tekhnicheskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki **221**, 684–698 (2017).

[27] A. Manzhirov, "Some problems in mechanics of growing solids with applications to am technologies," in *Journal of Physics: Conference Series*, Vol. 991 (IOP Publishing, 2018) p. 012056.

[28] N. Stadnik and E. Dats, "Continuum mathematical modelling of pathological growth of blood vessels," in *Journal of Physics: Conference Series*, Vol. 991 (IOP Publishing, 2018) p. 012075.