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Deformation and Heating of an Elastoviscoplastic Cylindrical Layer Moving Owing to a Varying Pressure Drop

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Abstract—Within the framework of the theory of large elastoplastic deformations generalized to the case of viscous and thermophysical properties of materials, we give a solution of a sequence of coupled problems on the onset and development of a flow in a material layer filling the gap between two rigid coaxial cylindrical surfaces under increasing pressure drop and on the subsequent flow deceleration under decreasing pressure gradient. Here the thermophysical and deformation processes are coupled, and the yield stress depends on temperature. Heat production due to the layer material friction against the rough cylindrical boundary surfaces is taken for an additional heat source.

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1. INTRODUCTION

The problem analyzed in this article is one of a number of problems that technological practice poses before fundamental mechanics. The statement of such a problem has arisen from the need to model, for example, the processes occurring in the material during high-velocity forming, high-temperature pressing in powder metallurgy, pressing models in high-precision casting [1]. In all these processes, deformations acquired by materials are large. Along with the elastic properties of materials, plastic and viscous properties must be taken into account. At the same time, the material being processed is noticeably heated both by deformation and by friction against the rigid walls. Therefore, the problem turns out not to be isothermal and is considered within the framework of the theory of large deformations of media with elastic, plastic, and viscous properties. We assume that the yield stress depends on temperature, but the deformation, heat release, and heat transfer are not separated. Note that currently there are few publications dealing with the coupled problems of the theory of large elastoviscoplastic deformations [2–6]. For the mathematical model of large deformations we take the model proposed earlier in [7] and generalized to the nonisothermal case in [8, 9].

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2. BASIC MODEL RELATIONS

In the rectangular spatial Euler coordinate system x_i, the kinematics of large deformations of the medium [9] is determined by the dependences

$$
\begin{aligned}
& d_{ij} = m_{ij} + p_{ij} - \frac{1}{2} m_{ik} m_{kj} - m_{ik} p_{kj} - p_{ik} m_{kj} + m_{ik} p_{km} m_{mj}, \\
& \frac{Dm_{ij}}{Dt} = \varepsilon_{ij} - \varepsilon^{\mathrm{p}}_{ij} - \frac{1}{2}[(\varepsilon_{ik} - \varepsilon^{\mathrm{p}}_{ik} + z_{ik}) m_{kj} + m_{ik}(\varepsilon_{kj} - \varepsilon^{\mathrm{p}}_{kj} - z_{kj})], \\
& \frac{Dp_{ij}}{Dt} = \varepsilon^{\mathrm{p}}_{ij} - p_{ik}\varepsilon^{\mathrm{p}}_{kj} - \varepsilon^{p}_{ik} p_{kj}, \quad \frac{Dn_{ij}}{Dt} = \frac{dn_{ij}}{dt} - r_{ik} n_{kj} + n_{ik} r_{kj}, \\
& \varepsilon_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}), \quad v_i = \frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_{i,j} v_j, \quad u_{i,j} = \frac{\partial u_i}{\partial x_j}, \\
& m_{ij} = e_{ij} + \alpha(T - T_0)\delta_{ij}, \quad r_{ij} = \frac{1}{2}(v_{i,j} - v_{j,i}) + z_{ij}(\varepsilon_{sk}, m_{sk}).
\end{aligned} \tag{2.1}
$$

In relations, (2.1) d_{ij} are the components of the Almansi strain tensor, $m_{ij} - \frac{1}{2} m_{ik} m_{kj}$ and p_{ij} are its reversible and irreversible components; D/Dt is the used objective time derivative of tensors reduced for an arbitrary tensor n_{ij}; $\varepsilon^{\mathrm{p}}_{ij}$ are the plastic strain rate tensor components, r_{ij} are the components of the rotation tensor; z_{ij} is the nonlinear component of the rotation tensor, which was completely written out in [9]; v_i and u_i are the components of the vectors of velocities and displacements of medium points. The thermal expansion is considered reversible, thus, e_{ij} is the linear part of the elastic strain tensor, α is the coefficient of linear expansion, T is the current temperature, T_0 is the room temperature of the body in the free state.

We accept the condition that the volume change of the medium is determined only by its thermal expansion and it is mechanically incompressible. Then we have

$$
\begin{aligned}
& \sigma_{ij} = \begin{cases} -P\delta_{ij} + (1 + 3\alpha T_0 \theta)^{-1} \dfrac{\partial W}{\partial d_{ik}}(\delta_{kj} - 2d_{kj}) & \text{for } p_{ij} \equiv 0, \\ -P_1\delta_{ij} + (1 + 3\alpha T_0 \theta)^{-1} \dfrac{\partial W}{\partial m_{ik}}(\delta_{kj} - m_{kj}) & \text{for } p_{ij} \neq 0, \end{cases} \\
& \theta = (T - T_0) T_0^{-1}.
\end{aligned} \tag{2.2}
$$

In the formulas (2.2) σ_{ij} are the components of the Euler-Cauchy stress tensor, P and P_1 are the unknown functions of the additional hydrostatic pressure. In deriving relations (2.2) we used the assumption that the density of the free energy distribution ψ depends only on reversible deformations, thus, we have $W = \rho_0 \psi$ (ρ_0 is the density of the material in the free state). For the elastic potential W we take its expansion in the Maclaurin series with respect to the free state at temperature T_0. Assuming the isotropy condition, we obtain

$$
W = -2\mu J_1 - \mu J_2 + bJ_1^2 + (b - \mu) J_1 J_2 - \chi J_1^3 + \nu_1 J_1 \theta + \nu_2 \theta^2 - \nu_3 J_1 \theta^2 - \nu_4 J_1^2 \theta - \nu_5 J_2 \theta - \nu_6 \theta^3 + \cdots, \tag{2.3}
$$

$$
J_k = \begin{cases} L_k & \text{for } p_{ij} = 0, \\ I_k & \text{for } p_{ij} \neq 0, \end{cases} \quad L_1 = d_{kk}, \quad L_2 = d_{ik} d_{ki}, \quad I_1 = c_{kk}, \quad I_2 = c_{ik} c_{ki}, \quad c_{ij} = m_{ij} - \tfrac{1}{2} m_{ik} m_{kj}.
$$

Here μ is the shear modulus, b, χ, ν_m ($m = 1, 2, \ldots, 6$) are the thermomechanical constants [10].

Taking into account the Fourier's law and according to the entropy balance equation, we obtain the heat conduction equation (q is the thermal diffusivity coefficient):

in the region of reversible deformation

$$
\begin{aligned}
& (1 + \beta_1 \theta + \beta_2 d_{jj}) \frac{\partial \theta}{\partial t} + \beta_3 \varepsilon_{ij} d_{ji} = q \frac{\partial^2 \theta}{\partial x_j \partial x_j}, \\
& \beta_1 = \frac{\nu_2(1 - 3\alpha T_0) - 3\nu_6}{\nu_2}, \quad \beta_2 = -\frac{\nu_3}{\nu_2}, \quad \beta_3 = -\frac{\nu_1 + \nu_5}{\nu_2};
\end{aligned} \tag{2.4}
$$

in the flow region

$$
(1 + \beta_1 \theta + \beta_2 c_{jj}) \frac{\partial \theta}{\partial t} + \beta_3 (\varepsilon_{ij} - \varepsilon^{\mathrm{p}}_{ij}) c_{ji} = q \frac{\partial^2 \theta}{\partial x_j \partial x_j} - \frac{1}{2\nu_2} \sigma_{ij} \varepsilon^{\mathrm{p}}_{ij}; \tag{2.5}
$$