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Thermal Stresses in an Elastoplastic Tube Depending on the Choice of Yield Conditions

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Abstract—We use the solution of a one-dimensional problem of the theory of thermal stresses in an elastoplastic tube heated on its interior surface and maintained at a constant temperature on the exterior surface as an example to make a comparison of both the results and solution methods depending on the choice of each of three conventional yield criteria: piecewise linear criteria of maximum shear and maximum reduced stresses and a smooth criterion of maximum octahedral stresses. It is established that while the transition of stresses from the face of the Tresca prism to its edge (change in the flow regime) in the first of the piecewise linear yield criteria takes place at the plastic flow onset, in the second one, this transition occurs on the elastoplastic boundary. The yield stress is assumed to be temperature dependent.

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1. INTRODUCTION

Within the framework of the plastic flow theory, the solution of the boundary value problem on thermal stresses in a thick-walled tube made from an ideal elastoplastic material was first obtained by D. Bland [1]. The possibility of obtaining such an analytical solution has been determined mainly by using Tresca-Saint Venant piecewise linear yield criterion. It has been observed that in this case the region of plastic deformation can be divided into parts, in which irreversible deformation occurs in accordance with different systems of differential equations depending on the belonging of stressed states to different faces and edges of the yield surface (the inclined Tresca prism in the 3D space of principal stresses). Thus, the piecewise linear yield criteria simplifying the mathematical apparatus with the possibility of obtaining an analytical solution give rise to another difficulty associated with the plastic region separation into parts. The time instants and onset places of boundary surfaces separating these plastic subregions together with the place and time of onset of elastoplastic boundaries as well as the consistent patterns of advancement of these surfaces along the deforming material are the integral elements of solutions being built and, therefore, they must be monitored in the process of constructing solutions. This is especially necessary when the process of thermomechanical action ends with the unloading and cooling to room temperature. This happens, for example, when modeling the technological operation of assembling elastoplastic cylindrical parts by the shrink fit method [2–5].

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If we assume that the mechanical parameters (elastic moduli, yield stress) depend on temperature then it becomes more complicated to obtain analytical solutions. Sometimes it is even impossible, since the statement of the problem using piecewise linear yield criteria, for example, the Tresca-Saint Venant criterion of the maximum shear stresses is contradictory, despite the fact that the solution exists for a constant yield stress. Changes in the geometry of the calculated regions of plastic flow and especially the appearance and disappearance of these regions also make it difficult to approximate numerical calculations, therefore, active processes are mainly considered without taking into account the unloading [6–10]. In this article, we also consider, in the framework of the theory of thermal stresses, only the active thermal loading of a thick-walled elastoplastic tube, the yield stress of which depends on temperature. The main objective here is to compare both the results of the calculations of the acquired irreversible deformations and thermal stresses and methods for calculating them.

2. STATEMENT OF THE PROBLEM. THE INITIAL EQUATIONS

Assume that the tube is made of an elastoplastic material limited by cylindrical surfaces $r = R_1$ (internal radius) and $r = R_2$ (external radius). Thermal action on the material is associated with a proportional to time increase of temperature on its internal surface, while the external surface is maintained at a constant room temperature T_0. We suppose that until the start of heating $t = 0$ the material is in a free state at the temperature T_0. In order to obtain analytical solutions, we assume that the temperature increase is slow and the temperature distribution corresponds to the quasi-stationary case

$$T(r,t) - T_0 = \psi t \frac{\ln(r/R_2)}{\ln(R_1/R_2)}. \tag{2.1}$$

Here ψ is the heating rate. The lateral surfaces of the tube are considered to be free: $\sigma_{rr}(R_1) = 0$, $\sigma_{rr}(R_2) = 0$. Deformations in the tube material are assumed to be small and formed by reversible (elastic) and irreversible (plastic) deformations. Consequently, under the conditions of one-dimensional problem in a cylindrical coordinate system (r, φ, z) we have the relations:

$$d_{rr} = e_{rr} + p_{rr} = u_{r,r}, \quad d_{\varphi\varphi} = e_{\varphi\varphi} + p_{\varphi\varphi} = r^{-1}u_r, \quad d_{zz} = e_{zz} + p_{zz} = 0. \tag{2.2}$$

Here d_{rr}, $d_{\varphi\varphi}$, d_{zz} are the complete deformations, e_{rr}, $e_{\varphi\varphi}$, e_{zz} are the reversible deformations, p_{rr}, $p_{\varphi\varphi}$, p_{zz} are the irreversible components of deformation. Reversible deformations specify the stresses in the tube material [11]

$$\begin{aligned} \sigma_{rr} &= (\lambda + 2\mu)e_{rr} + \lambda(e_{\varphi\varphi} + e_{zz}) - 3K\theta, \\ \sigma_{\varphi\varphi} &= (\lambda + 2\mu)e_{\varphi\varphi} + \lambda(e_{rr} + e_{zz}) - 3K\theta, \\ \sigma_{zz} &= (\lambda + 2\mu)e_{zz} + \lambda(e_{rr} + e_{\varphi\varphi}) - 3K\theta. \end{aligned} \tag{2.3}$$

In (2.3) λ and μ are the elastic Lame constants, $K = \lambda + 2\mu/3$ is the bulk modulus, $\theta = \alpha(T - T_0)$ is the thermal component of deformations, and α is the coefficient of thermal expansion. With temperature growth, elastic deformations and, consequently, stresses increase. Irreversible deformation begins with the emergence of the stressed states on the yield surface, that is, when the plastic flow condition is satisfied. As such conditions, conventional yield criteria [12, 13] will be used below:
the criterion of maximum shear stress (the Tresca-Saint Venant criterion):

$$f(\sigma_1, \sigma_2, \sigma_3) = \max|\sigma_i - \sigma_j| - 2k = 0; \tag{2.4}$$

the maximum octahedral stress criterion (the von Mises criterion)

$$f(\sigma_1, \sigma_2, \sigma_3) = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - 8k^2 = 0; \tag{2.5}$$

criteria of maximum reduced stress (the Ishlinsky–Ivlev criterion):

$$f(\sigma_1, \sigma_2, \sigma_3) = \max|\sigma_i - \sigma| - \tfrac{3}{4}k = 0, \quad \sigma = \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3). \tag{2.6}$$

Here σ_i are the principal values of the stress tensor, k is the yield stress.

For the latter we use its simplest temperature dependence

$$k(\theta) = k_0(1 - \beta\theta), \tag{2.7}$$