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On Heating of Thin Circular Elastic-Plastic Plate with the Yield Stress Depending on Temperature

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Abstract

The present paper deals with the problem of the elastic-plastic plate heating. Considering problems are solved with the various yield stress depending on temperature. Throughout the paper the model of thermo-elastic-plastic deformation are used. We consider Tresca yield criterion, von Mises one, and Ishlinskiy-Ivlev one. The boundaries of the irreversible deformation domain are computed according to the analytical and numerical results. A comparison of the analytical results for various yield stresses is discussed. The residual stresses were computed and graphically analyzed. The characteristics of the plastic flow in the heating domain according to the yield strength selection are eliminated.

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Keywords:

1. Main text

Thermal stresses have a significant effect on parts of various mechanisms operating under high temperature gradients. Nonstationary temperature field variations result in the formation of residual strains and stresses. Accounting of such strains and stresses is necessary for accurate determination of the geometry and strength characteristics of the related objects. It is well known that temperature affects the yield stress of material by increasing the probability of appearance of irreversible deformations.

The boundary value problem of the heating and cooling of elastic-plastic bodies previously considered in [1-17]. For example, in [1] the analytical solution was found for the Tresca yield criterion in problems of the plastic flow. It

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was shown that the irreversible deformation region composed of several parts. The analytical solution for the central heating of a thin circular plate with Ishlinsky-Ivlev yield criterion depending on temperature was obtained in [3]. It has been found that the assumption of the Tresca yield criterion leads to physically incorrect results in the irreversible deformation domain. The results obtained in [3] are valid for the case of plane strain condition. Yield strength is constitutive material parameter and strongly depends on temperature. Therefore, the such dependence assuming in the thermal plasticity problems allows to obtain more faithful solutions in the plastic domains, as well as to identify the differences in solutions for different yield criteria. In this paper, we solved one-dimensional problem of long hollow cylinder heating by irregular thermal field under Ishlinsky-Ivlev yield criterion.

The boundary value problem for a radial symmetrical stresses in a perfectly plastic disk heated by a heat source of circular shape and constant output was investigated in [14]. At present study we consider rapid heating and consequence cooling of a thermos-elastic-plastic disk. The analytical solution for stress-strain state under unsteady temperature gradient was obtained due to Tresca yield criterion with temperature independent yield stress. In [15] the problem of circular contour heating of a plate under increasing temperature gradient was discussed. It was shown that at high temperature levels the plastic flow domain reaches a size at which it becomes possible existence of additional plastic flow domains with the different edges of Tresca yield criterion. The problem in the Tresca yield criterion framework, taking into account the linear dependence of the yield stress on temperature was first considered in [3]. This paper presents a new solution to the problem with Ishlinsky-Ivlev yield criterion [16].

2. Governing equations. Thermoelastic equilibrium.

Let consider an infinite thin plate which is rapidly heating at a predetermined radius $r = R$. The boundary conditions for temperature are given in the form:

$$T(R,t) - T_0 = T_m(1-\exp(-xt)), \quad \left.\frac{\partial T}{\partial r}\right|_{r=0} = 0, \quad \left.\frac{\partial T}{\partial r}\right|_{r=\infty} = 0. \tag{1}$$

where x is the temperature balancing speed, T_m, T_0 are the maximal and referential temperature. The analytical solution for the given temperature field (Fig. 1a) is presented in [3]. Material of the plate assume like the elastic-plastic media. Infinitisimal strains consist of reversible e_{ij} (thermoelastic) and irreversible p_{ij} (plastic) strains:

$$\frac{u_r}{r} = e_{\varphi\varphi} + p_{\varphi\varphi}, \quad \frac{\partial u_r}{\partial r} = e_{rr} + p_{rr}, \tag{2}$$

wherein u_r is the radial component of the displacement vector.

The relation between the components of the stress tensor and elastic deformation is defined by using the Duhamel-Neumann law [17]:

$$\sigma_{rr} = \frac{4\mu(\lambda+\mu)e_{rr}}{(\lambda+2\mu)} + \frac{2\lambda\mu e_{\varphi\varphi}}{(\lambda+2\mu)} - \frac{2\mu(3\lambda+2\mu)\Delta}{(\lambda+2\mu)}, \quad \sigma_{\varphi\varphi} = \frac{4\mu(\lambda+\mu)e_{\varphi\varphi}}{(\lambda+2\mu)} + \frac{2\lambda\mu e_{rr}}{(\lambda+2\mu)} - \frac{2\mu(3\lambda+2\mu)\Delta}{(\lambda+2\mu)}. \tag{3}$$

Here λ, μ are the Lame parameters, $\Delta = \alpha(T - T_0)$ is the deformation of linear thermal expansion, α is thermal expansion coefficient. Constitutive equations (3) are supplemented by equilibrium equation and boundary condition

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = 0, \quad u_r(0,t) = 0, \quad u_r(\infty,t) = 0. \tag{4}$$

We obtain a closed system of differential equations determining the thermoelastic state of the material at a given thermal distribution. The solution of this system reads:

$$\sigma_{rr} = -\frac{\gamma}{r^2}\int_0^r \Delta(\rho,t)\rho d\rho\ , \quad \sigma_{\varphi\varphi} = \frac{\gamma}{r^2}\int_0^r \Delta(\rho,t)\rho d\rho - \gamma\Delta(r,t)\ , \quad u_r = \frac{\gamma}{2\mu r}\int_0^r \Delta(\rho,t)\rho d\rho\ . \tag{5}$$

3. Plastic Flow and Unloading.

The plastic flow begun from a certain temperature level due to Ishlinsky-Ivlev yield criterion satisfaction

$$\sigma_{rr}(R,t) - 2\sigma_{\varphi\varphi}(R,t) = 4k(R,t) \tag{6}$$

Here $k(r,t) = k_0(1-\beta\Delta(r,t))$ is the linear function on temperature, k_0 referential yield stress, β is the degree of the yield stress dependence on temperature. According to plastic flow rule [16], associated with yield surface (6). Thus the plastic incompressibility is valid for the plastic strain increment:

$$p_{rr} + p_{\varphi\varphi} + p_{zz} = 0\ , \quad p_{rr} = p_{zz}\ . \tag{7}$$

We obtain the function of thermal stresses, the radial displacement and plastic deformations in the plastic flow domain $a < r < b$ by integrating the system of equations (4), (6), (7) in forms:

$$\begin{gathered}
\sigma_{rr} = -\frac{2}{\sqrt{r}}\int_a^r \frac{k(\rho,t)}{\sqrt{\rho}}d\rho + \frac{X(t)}{\sqrt{r}}, \quad \sigma_{\varphi\varphi} = -\frac{1}{\sqrt{r}}\int_a^r \frac{k(\rho,t)}{\sqrt{\rho}}d\rho + \frac{X(t)}{2\sqrt{r}} - 2k(r,t), \quad \varsigma = \frac{\mu(3\lambda+2\mu)}{(3\lambda+5\mu)}, \\
u_r = \frac{1}{2}\left(\frac{3}{\gamma\sqrt{r}}\int_a^r k(\rho,t)\sqrt{\rho}d\rho - \frac{\sqrt{r}}{\varsigma}\int_a^r \frac{k(\rho,t)}{\sqrt{\rho}}d\rho + \frac{3}{\sqrt{r}}\int_a^r \Delta(\rho,t)\sqrt{\rho}d\rho + \frac{\sqrt{r}X(t)}{2\varsigma} + \frac{Y(t)}{\sqrt{r}}\right), \\
p_{rr} = \frac{3}{4\sqrt{r}}\left(\frac{1}{\gamma}\int_a^r \frac{k(\rho,t)}{\sqrt{\rho}}d\rho - \frac{1}{\gamma r}\int_a^r k(\rho,t)\sqrt{\rho}d\rho - \frac{1}{r}\int_a^r \Delta(\rho,t)\sqrt{\rho}d\rho\right) - \frac{k(r,t)}{\gamma} - \frac{3X(t)}{8\gamma\sqrt{r}} - \frac{Y(t)}{4r\sqrt{r}} + \frac{\Delta(r,t)}{2}.
\end{gathered} \tag{8}$$

Herein X, Y are the unknown functions. The thermoelastic equilibrium occurs in two areas separated by the plastic flow domain. Thus for each thermoelastic domains one can obtained due to (5) two similar solution:

$$\begin{gathered}
\sigma_{rr} = A(t) - \frac{\gamma}{r^2}\int_0^r \Delta(\rho,t)\rho d\rho, \quad \sigma_{\varphi\varphi} = A(t) + \frac{\gamma}{r^2}\int_0^r \Delta(\rho,t)\rho d\rho - \gamma\Delta(r,t), \quad u_r = \frac{\gamma}{2\mu r}\int_0^r \Delta(\rho,t)\rho d\rho + \frac{(\lambda+2\mu)A(t)r}{\mu(3\lambda+2\mu)}, \\
\sigma_{rr} = \frac{B(t)}{r^2} - \frac{\gamma}{r^2}\int_b^r \Delta(\rho,t)\rho d\rho, \quad \sigma_{\varphi\varphi} = -\frac{B(t)}{r^2} + \frac{\gamma}{r^2}\int_b^r \Delta(\rho,t)\rho d\rho - \gamma\Delta(r,t), \quad u_r = \frac{\gamma}{2\mu r}\int_b^r \Delta(\rho,t)\rho d\rho - \frac{B(t)}{2\mu r}.
\end{gathered} \tag{9}$$

Unknown functions contained in the equations (8), (9) are derived from the boundary conditions (4) and the stress and displacements continuity on the elastic-plastic boundaries:

$$
\begin{aligned}
A &= \frac{3}{(9b-a)\sqrt{a}}\left(6\int_a^b \frac{k(\rho,t)}{\sqrt{\rho}}d\rho - 2\int_a^b k(\rho,t)\sqrt{\rho}d\rho - 2\gamma\int_a^b \Delta(\rho,t)\sqrt{\rho}d\rho + \frac{3(b-a)\gamma}{a\sqrt{a}}\int_0^a \Delta(\rho,t)\rho d\rho\right),\\
B &= \frac{2b\sqrt{b}}{(9b-a)}\left(a\int_a^b \frac{k(\rho,t)}{\sqrt{\rho}}d\rho - 3\int_a^b k(\rho,t)\sqrt{\rho}d\rho - 3\gamma\int_a^b \Delta(\rho,t)\sqrt{\rho}d\rho - \frac{4\gamma}{\sqrt{a}}\int_0^a \Delta(\rho,t)\rho d\rho\right),\\
X &= \frac{2}{(9b-a)}\left(9b\int_a^b \frac{k(\rho,t)}{\sqrt{\rho}}d\rho - 3\int_a^b k(\rho,t)\sqrt{\rho}d\rho - 3\gamma\int_a^b \Delta(\rho,t)\sqrt{\rho}d\rho - \frac{4\gamma}{\sqrt{a}}\int_0^a \Delta(\rho,t)\rho d\rho\right),\\
Y &= \frac{2}{(9b-a)}\left(9b\int_a^b \frac{k(\rho,t)}{\sqrt{\rho}}d\rho - 3\int_a^b k(\rho,t)\sqrt{\rho}d\rho - 3\gamma\int_a^b \Delta(\rho,t)\sqrt{\rho}d\rho - \frac{4\gamma}{\sqrt{a}}\int_0^a \Delta(\rho,t)\rho d\rho\right).
\end{aligned} \tag{10}
$$

The position of elastic-plastic boundaries a , b is calculated by numerical solution of two equations $p_{rr}(a,t)=0$, $p_{rr}(b,t)=0$.

One can be found that the temperature field can reach values for which the yield criterion edge may be chahged from conditions (6) to the conditions $\sigma_{rr} - 2\sigma_{\varphi\varphi} = 4k$, $\sigma_{rr} + \sigma_{\varphi\varphi} = -4k$. Thus the equilibrium equation transforms to $\frac{k}{r} = \frac{\partial k}{\partial r}$. Obviously, this equation can be satisfied only if the yield strength is a linear depend on a radius and if a heat flux is stationary linear. Consequently, the parameter β is essential with uneven heat distribution. Note that the one-dimensional elastic-plastic problem has no solutions at low yield strength in the cylindrical symmetry case.

Consider the process of the material unloading. The analytical solution for the temperature during cooling (Fig. 1b) is discussed in [3]. Assume that the irreversible deformation during the cooling under fixing irreversible deformation at some time $t = t_u$ gives by

$$
\begin{cases}
p'_{rr}(r) = 0 \quad r < a,\\
p'_{rr}(r) = p_{rr}(r,t_u) \quad r \in [a,b],\\
p'_{rr}(r) = 0 \quad r > b,
\end{cases} \tag{11}
$$

This notation becomes useful in deriving of the analytical relations describing the stress-strain state in the process of cooling (unloading).

We write the solutions for the stress (5) based on the irreversible deformation (11) and the boundary conditions (4) in form

$$
\begin{aligned}
\sigma_{rr} &= \gamma\left(\frac{3}{2}\int_0^r \frac{p'_{rr}(\rho)}{\rho}d\rho + \frac{1}{r^2}\int_0^r p'_{rr}(\rho)\rho d\rho - \frac{3}{2}\int_a^b \frac{p'_{rr}(\rho)}{\rho}d\rho - \frac{1}{r^2}\int_0^r \Delta(\rho,t)\rho d\rho\right),\\
\sigma_{\varphi\varphi} &= \gamma\left(\frac{3}{2}\int_0^r \frac{p'_{rr}(\rho)}{\rho}d\rho - \frac{1}{r^2}\int_0^r p'_{rr}(\rho)\rho d\rho - \frac{3}{2}\int_a^b \frac{p'_{rr}(\rho)}{\rho}d\rho + \frac{1}{r^2}\int_0^r \Delta(\rho,t)\rho d\rho + 2p'_{rr}(r) - \Delta(r,t)\right).
\end{aligned} \tag{12}
$$

The terms in equation (11) containing the linear thermal expansion Δ should be vanished to obtain relations for the residual stresses.

4. Discussion.

The distribution of thermal stress during the plastic flow and after full cooling are shown on Fig. 2. The vertical dashed lines indicate the plastic flow boundaries. We note one more feature of the irreversible deformations formation in the plastic flow domain. The rate of plastic deformation equal to zero during the process of reducing the temperature, which signifies the beginning of the unloading process under heating. This fact is neglected, since it can be concluded that the rapid transient heating with subsequent unloading in the subsequent alignment of the temperature field slightly changes the final residual deformation on the basis of a number of previously solved problems [4, 5]. Fig. 3 shows the results of thermal stress calculations at constant and variable yield strength. We use the numerical solution for von Mises yield criterion and analytical solution for the Tresca yield criterion [3] to compare the obtained analytical results. Additionally, for the conditions Tresca may be valid the effect of "repeated plastic flow", when in the process of cooling the level of residual stress generated is large enough the irreversible deformation, in which the increment of plastic strain occurs with the opposite sign. The repeated plastic flow occurs immediately after unloading the material for a certain values of β during the heating under Ishlinsky-Ivlev yield criterion framework. For these case Ishlinsky-Ivlev yield criterion transforms to $\sigma_{rr} + \sigma_{\varphi\varphi} = -4k$. The present solution of the problem shows that the yield point dependence on temperature can lead to the edges change in Ishlinsky-Ivlev yield criterion during unsteady heating of the material, even in plane stress frameworks. Furthermore, the problem could not have a solution for a specific yield strength dependence on temperature. This fact certainly should be considered for the correct setting and more accurate solutions of the thermoelastic-plastic problems.

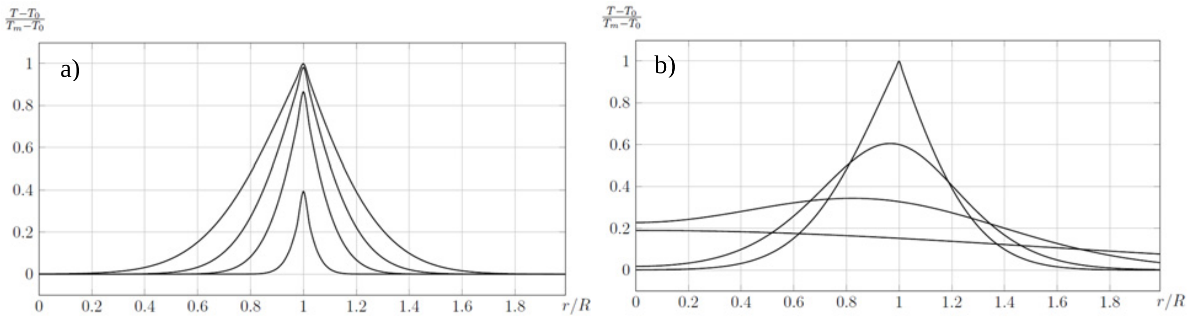

Fig. 1. Temperature field: a) Heating, b) Cooling.

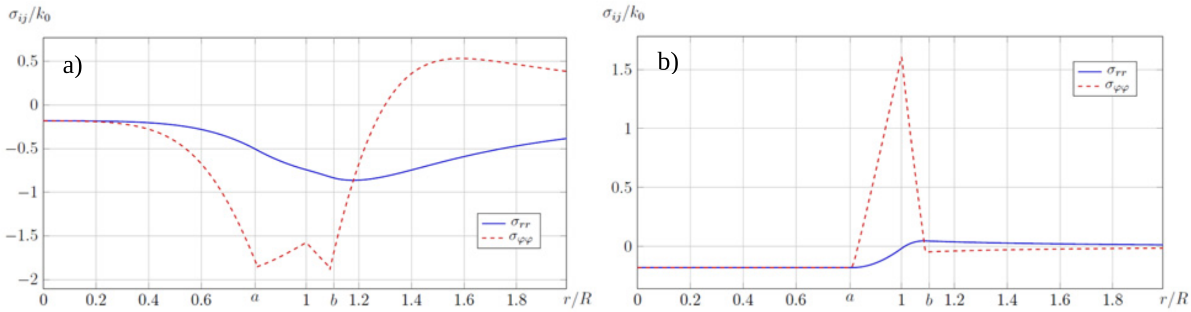

Fig. 2. Thermal stresses: a) Plastic flow, b) Unloading (residual stresses).
$a/R = 0.814$, $b/R = 1.091$, $k_0 = 210\times10^6\,Pa$, . $\Delta(R) = 4.25\times10^{-3}$, $\beta = 94.12$, , $\mu = 4.29\times10^{10}\,Pa$, $R = 0.2m$.

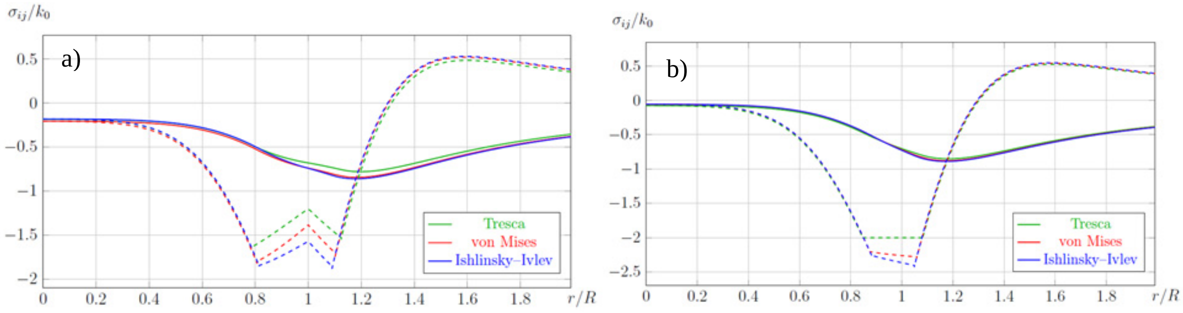

Fig. 3. Thermal stresses during plastic flow: a) $k = k(r,t)$, b) $k = const$.

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